FRAGMENTATION OF DISCS AROUND MASSIVE STARS

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PLAN

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THE EXISTENCE OF EXTENDED MASSIVE DISCS

To form a disc with radius R > 200 AU only requires the collapse of a prestellar core with

$$\beta \;\equiv\; \frac{U_{\rm ROT}}{|\Omega|} \;\gtrsim\; 0.06\,\left(\frac{M_{\rm CORE}}{10\,{\rm M}_\odot}\right)\;;$$

Alternatively, a protostar of mass M_1 must capture material with specific angular momentum

$$h \;\gtrsim\; h_{\rm MIN} \;\sim\; 2\times 10^{21}\,{\rm cm}^2\,{\rm s}^{-1}\,\left(\frac{M_1}{10\,{\rm M}_\odot}\right)^{1/2}$$

$$\equiv\; \left(0.2\,{\rm km\,s}^{-1}\right)\,\left(0.03\,{\rm pc}\right)\,\left(\frac{M_1}{10\,{\rm M}_\odot}\right)^{1/2}$$

It must happen pretty often.

DEFINITIONS

CENTRAL STAR

M_1, mass of star at centre of disc

L_1, luminosity of star at centre of disc

DISC

R, distance from central star

Σ, surface density of disc

a, sound speed

Ω, orbital angular speed

PROTOFRAGMENT

m, mass of proto-fragment

r, initial radius of proto-fragment

THE TOOMRE CONDITION

$$\ddot{r} = -\frac{Gm}{r^2} + \frac{1}{\rho}\frac{dP}{dr} + r\omega^2$$
$$= -\pi G\Sigma(R) + \frac{a^2(R)}{r} + \frac{r\Omega^2(R)}{4},$$

$$m = \pi r^2 \Sigma(R)$$

$$\frac{dP}{dr} \sim \frac{P}{r} = \frac{\rho a^2(R)}{r}$$

$$t_{\text{COND}} = \left(\frac{r}{-\ddot{r}}\right)^{1/2} = \left(\frac{\pi G\Sigma(R)}{r} - \frac{a^2(R)}{r^2} - \frac{\Omega^2(R)}{4}\right)^{-1/2},$$

$$t_{\text{FAST}} = 2\left(\left\{\frac{\pi G\Sigma(R)}{a(R)}\right\}^2 - \Omega^2(R)\right)^{-1/2}$$

Must be real, so $\Sigma(R) \gtrsim \dfrac{a(R)\Omega(R)}{\pi G}$.

$$a \propto R^{-1/4}$$
$$\Omega \propto R^{-3/2}.$$

THE GAMMIE CONDITION

A proto-fragment must be able to radiate away its compressional energy on a dynamical timescale, otherwise it undergoes an adiabatic bounce and is sheared apart.

$$t_{\rm COOL} \;\lesssim\; t_{\rm DYN}\,,$$

$$t_{\rm COOL} \;\simeq\; \frac{\Sigma(R)a^2(R)}{2\sigma_{\rm SB}T^4(R)/\Sigma(R)\bar{\kappa}_{\rm R}(R)}$$

$$t_{\rm DYN} \;\simeq\; \frac{2\pi}{\Omega(R)}$$

$$\Sigma(R) \;\lesssim\; \left(\frac{2^3\pi^4\bar{m}^{\,4}\,a^7(R)}{15Gc^2h^3\bar{\kappa}_{\rm R}(R)}\right)^{1/3}\,.$$

DUST OPACITY

$$\kappa_\lambda \propto \lambda^{-2} \longrightarrow \bar{\kappa}_{\rm R} \propto T^2 \longrightarrow \bar{\kappa}_{\rm R} \simeq \kappa_{\rm O}\, a^4$$

$$\kappa_{\rm O} \simeq 3\times 10^{-19}\,{\rm s^4\,cm^{-2}\,g^{-1}}$$

$$\Sigma(R) \lesssim \left(\frac{2^3\pi^4\bar{m}^{\,4}}{15Gc^2h^3\kappa_{\rm O}}\right)^{1/3} a(R)$$

GAMMIE CONDITION

$$\Sigma(R) \gtrsim \frac{a(R)\Omega(R)}{\pi G}$$

TOOMRE CONDITION

$$\Omega(R) \lesssim \left(\frac{2^3\pi^7G^3\bar{m}^{\,4}}{15c^2h^3\kappa_{\rm O}}\right)^{1/3}$$

GAMMIE + TOOMRE

$$R \gtrsim \left(\frac{15^2c^4h^6\kappa_{\rm O}^2{\rm M}_\odot^3}{2^6\pi^{14}G\bar{m}^8}\right)^{1/9}\left(\frac{M_1}{{\rm M}_\odot}\right)^{1/3} \sim 200\,{\rm AU}\left(\frac{M_1}{10\,{\rm M}_\odot}\right)^{1/3}$$

FRAGMENT PARAMETERS (DUST OPACITY)

$$a(R) \simeq 10^5\,\mathrm{cm\,s^{-1}} \left(\frac{L_1}{\mathrm{L_\odot}}\right)^{1/8} \left(\frac{R}{\mathrm{AU}}\right)^{-1/4}$$

$$\Omega(R) \simeq 2\times 10^{-7}\,\mathrm{s^{-1}} \left(\frac{M_1}{\mathrm{M_\odot}}\right)^{1/2} \left(\frac{R}{\mathrm{AU}}\right)^{-3/2}$$

radius

$$r_{\mathrm{FRAG}} \simeq \frac{2a(R)}{\Omega(R)} \gtrsim 100\,\mathrm{AU} \left(\frac{M_1}{10\,\mathrm{M_\odot}}\right)^{5/12} \left(\frac{L_1}{10^3\,\mathrm{L_\odot}}\right)^{1/8}$$

mass

$$m_{\mathrm{FRAG}} \simeq \frac{4a^3(R)}{G\Omega(R)} \gtrsim 0.08\,\mathrm{M_\odot} \left(\frac{M_1}{10\,\mathrm{M_\odot}}\right)^{1/6} \left(\frac{L_1}{10^3\,\mathrm{L_\odot}}\right)^{3/8}$$

MOLECULAR-LINE OPACITY (THE GAP)

$$\bar{\kappa}_{\rm R} \;\simeq\; \kappa_{\rm O} \left(\frac{\rho}{\rm g\,cm^{-3}}\right)^{2/3} \left(\frac{T}{\rm K}\right)^{3}, \qquad \kappa_{\rm O} \;\simeq\; 10^{-8}\,{\rm cm^2\,g^{-1}}$$

$$\Sigma(R) \;\lesssim\; \left(\frac{2^9\pi^{12}\bar{m}^{12}a^7(R)}{15^3G^3c^6h^9\kappa_{\rm O}^3}\right)^{1/13}$$

GAMMIE CONDITION

$$\Sigma(R) \;\gtrsim\; \frac{a(R)\Omega(R)}{\pi G}$$

TOOMRE CONDITION

$$\Omega^{13}(R)\,a^6(R) \;\lesssim\; \pi G\left(\frac{2^3\pi^8G^3\bar{m}^4}{15c^2h^3\kappa_{\rm O}}\right)^{3}$$

GAMMIE + TOOMRE

$$R \;\gtrsim\; 5\,{\rm AU}\left(\frac{M_1}{10\,{\rm M_\odot}}\right)^{13/42}\left(\frac{L_1}{10^3\,{\rm L_\odot}}\right)^{3/84}$$

FRAGMENT PARAMETERS (OPACITY GAP)

$$a(R) \;\simeq\; 10^5\,\mathrm{cm\,s^{-1}}\,\left(\frac{L_1}{\mathrm{L_\odot}}\right)^{1/8}\,\left(\frac{R}{\mathrm{AU}}\right)^{-1/4}$$

$$\Omega(R) \;\simeq\; 2\times 10^{-7}\,\mathrm{s^{-1}}\,\left(\frac{M_1}{\mathrm{M_\odot}}\right)^{1/2}\,\left(\frac{R}{\mathrm{AU}}\right)^{-3/2}$$

radius

$$r_{\mathrm{FRAG}} \;\simeq\; \frac{2a(R)}{\Omega(R)} \;\gtrsim\; 0.7\,\mathrm{AU}\,\left(\frac{M_1}{10\,\mathrm{M_\odot}}\right)^{-19/168}\,\left(\frac{L_1}{10^3\,\mathrm{L_\odot}}\right)^{57/336}$$

mass

$$m_{\mathrm{FRAG}} \;\simeq\; \frac{4a^3(R)}{G\Omega(R)} \;\gtrsim\; 0.003\,\mathrm{M_\odot}\,\left(\frac{M_1}{10\,\mathrm{M_\odot}}\right)^{-45/168}\,\left(\frac{L_1}{10^3\,\mathrm{L_\odot}}\right)^{135/336}$$

H_2-DISSOCIATION INSTABILITY

A protostar contracts quasistatically (Kelvin-Helmholtz) if (a) it is close to hydrostatic balance, $\Omega = -3PV$ (Virial Theorem), and (b) dynamical excursions are ``adiabatic''

A perfect gas has $P = nkT, \qquad u = n\,f\,\frac{kT}{2}\,,$

Hence $U = uV = \frac{f\,PV}{2}\,, \qquad \Omega = -\frac{6U}{f}\,.$

Therefore $L_\star = \frac{d\Omega}{dt} + \frac{dU}{dt} = \frac{d\Omega}{dt}\left\{1 - \frac{f}{6}\right\} = -0 \;\; \text{(2nd Law)}$

3 to 200 K,	f ~ 3,	3 translational only;
200 to 800 K,	f ~ 5,	add 2 rotational;
800 to 2000 K,	f ~ 7,	add 2 vibrational;
2000 to 6000 K,	f ~ 6,	dissociation (2 X 3).

CAVEATS

A) Since the discs are unstable, they may not have time to relax before fragmenting.

B) Magnetic effects may be important.

C) An impulsive perturbation (e.g. an infalling lump) will heat the disc locally, in the first instance suppressing fragmentation (e.g. Walch et al. 2009).

D) An impulsive perturbation may launch material directly into the non-linear regime of gravitational instability, but in so doing it will reduce the dynamical timescale, thereby exacerbating the Gammie condition (Whitworth et al. 2007, Clarke et al. 2007, Forgan & Rice 2009, etc.).

E) The cooling time due to convection is necessarily longer than the dynamical timescale, so convection does not help the Gammie condition significantly.

CONCLUSIONS

0. Massive, extended discs are probably hard to avoid, but they may be short-lived, because …

1. discs fragment outside a critical radius, R_{CRIT}.

2. For discs around Sun-like stars, dust opacity sets $R_{\text{CRIT}} \sim 100$ AU, and $m_{\text{FRAG}} \geq 0.003\ \text{M}_{\odot}$.

3. For discs around massive stars, dust opacity sets $R_{\text{CRIT}} \sim 200$ AU, and $m_{\text{FRAG}} \geq 0.08\ \text{M}_{\odot}$, and …

4. molecular-line opacity sets $R_{\text{CRIT}} \sim 1$ AU, and $m_{\text{FRAG}} \geq 0.003\ \text{M}_{\odot}$.

5. Fragmentation in the molecular-line opacity regime may be enhanced by H_2-dissociation.

6. More realistic simulations need to follow the formation of a disc (so it can at least have the opportunity to fragment before it relaxes)

7. Disc fragmentation is not promoted significantly by impulsive perturbations or convection.

8. Magnetic effects may significantly alter these conclusions.

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