Pressure assisted gravitational instability of expanding shells

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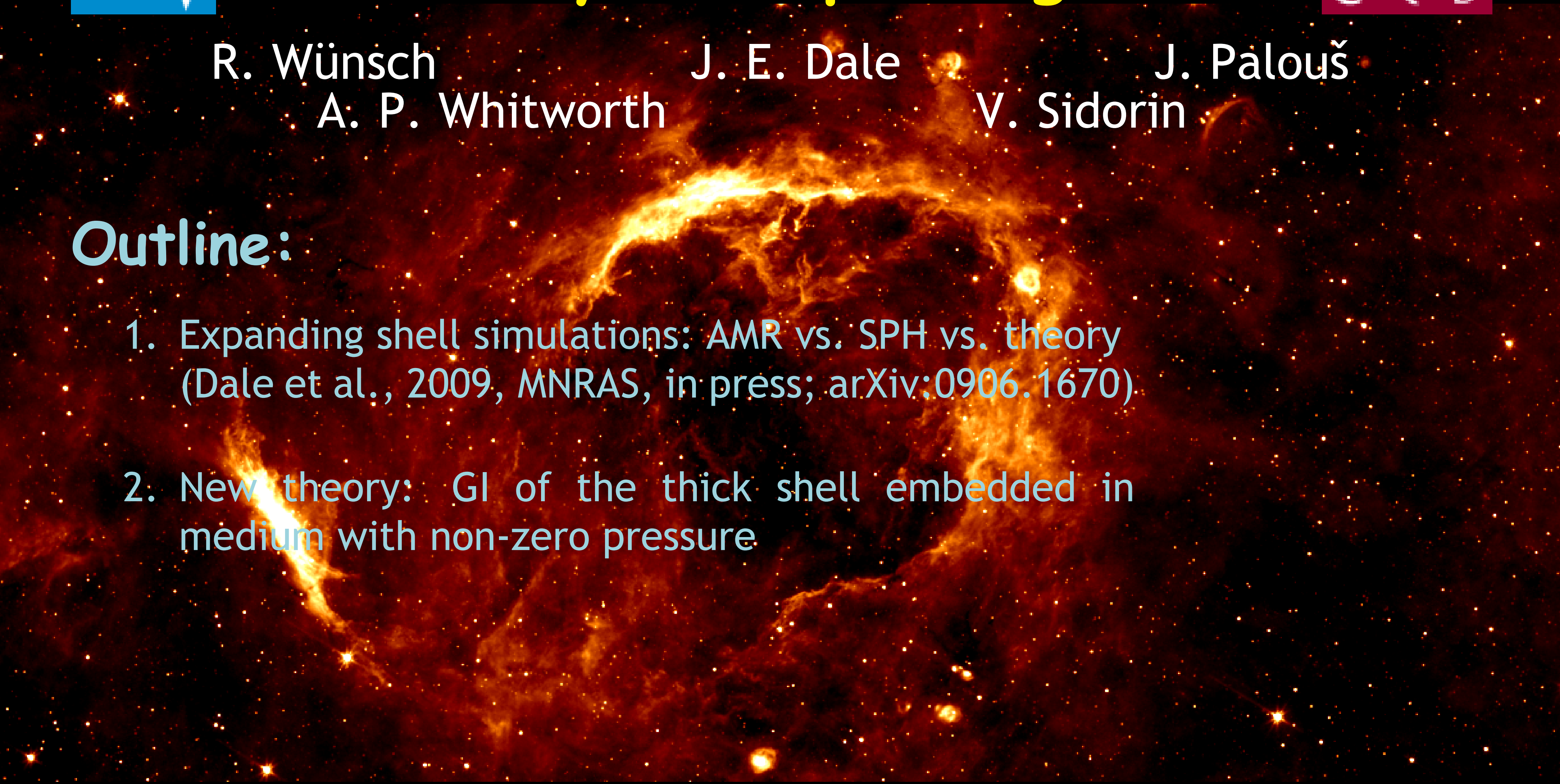
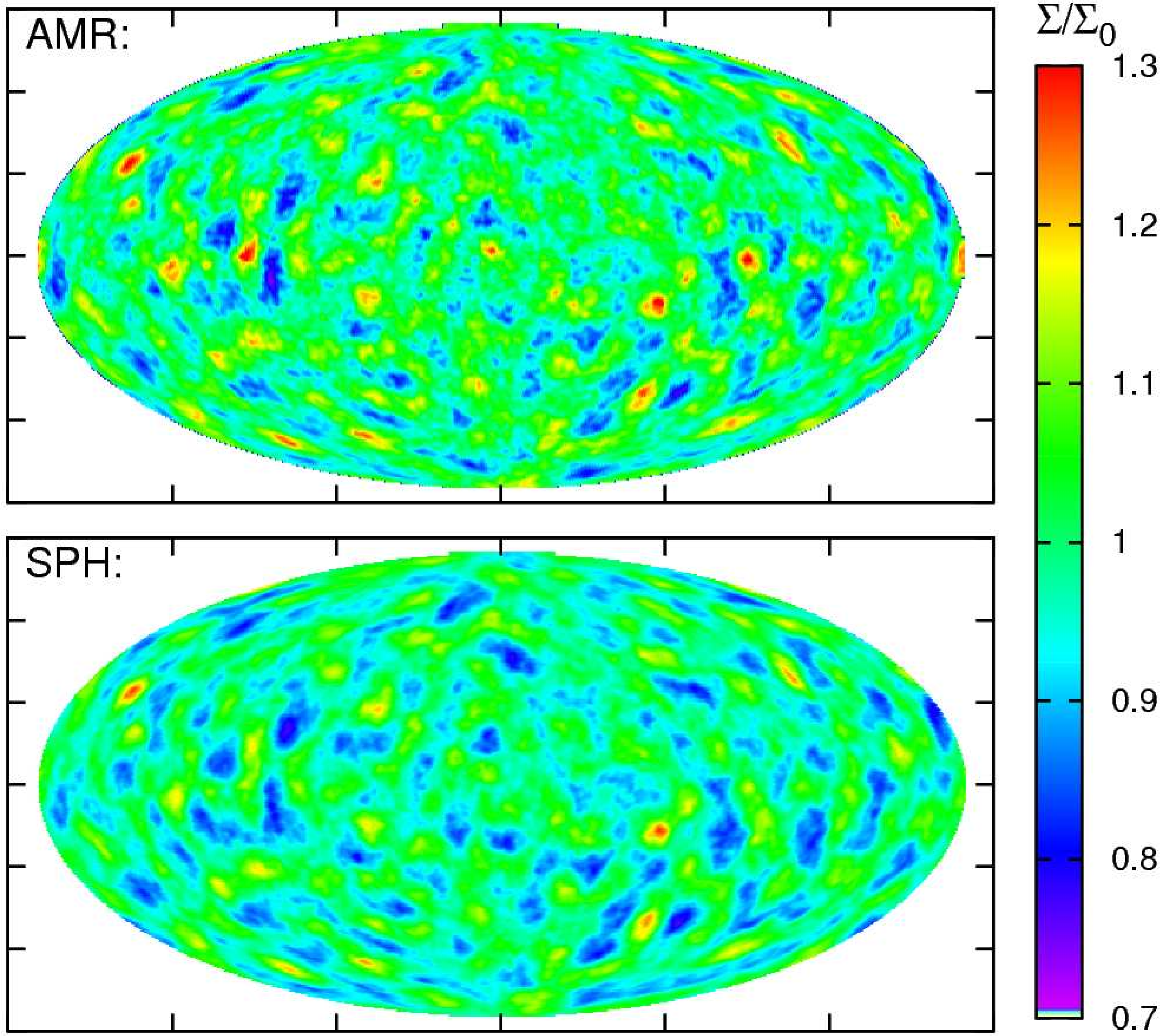
Outline:

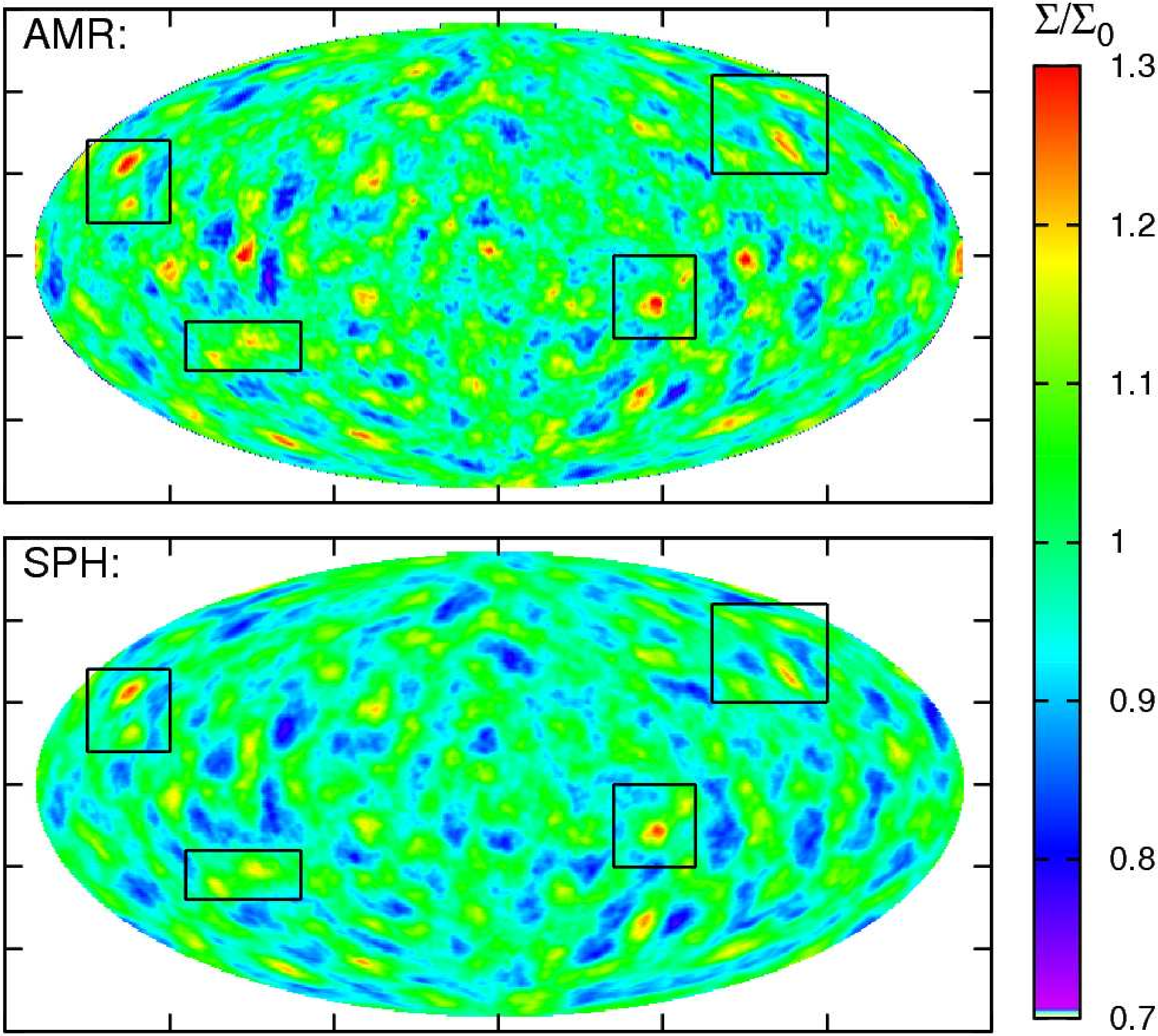
1. Expanding shell simulations: AMR vs. SPH vs. theory (Dale et al., 2009, MNRAS, in press; arXiv:0906.1670)

2. New theory: GI of the thick shell embedded in medium with non-zero pressure



Galactic bubble N107, Credit: Churchwell et al. (2006), Spitzer, GLIMPSE, IRAC, 8μm cont.


AMR:
SPH:
Σ/Σ0
1.3
1.2
1.1
1
0.9
0.8
0.7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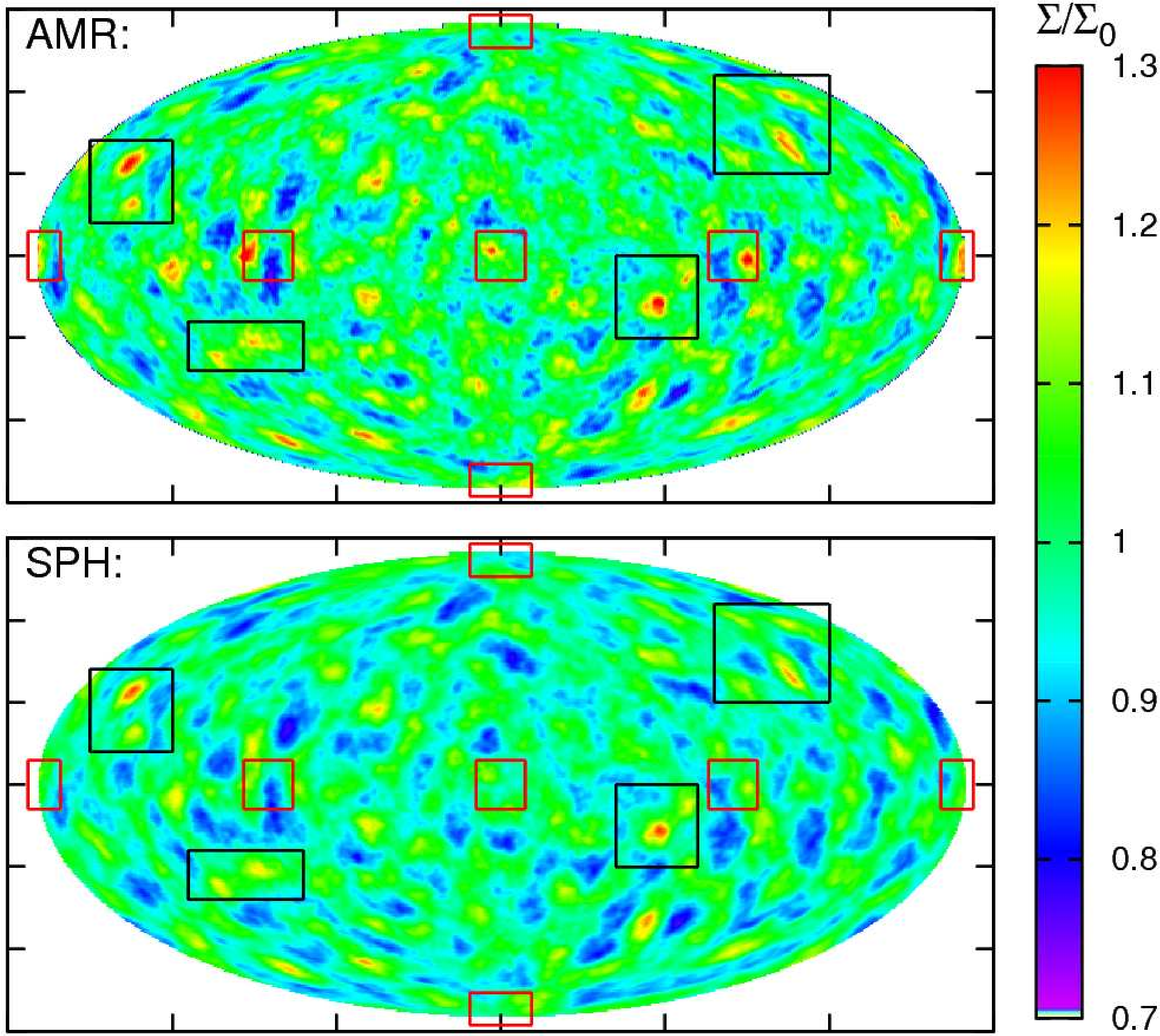
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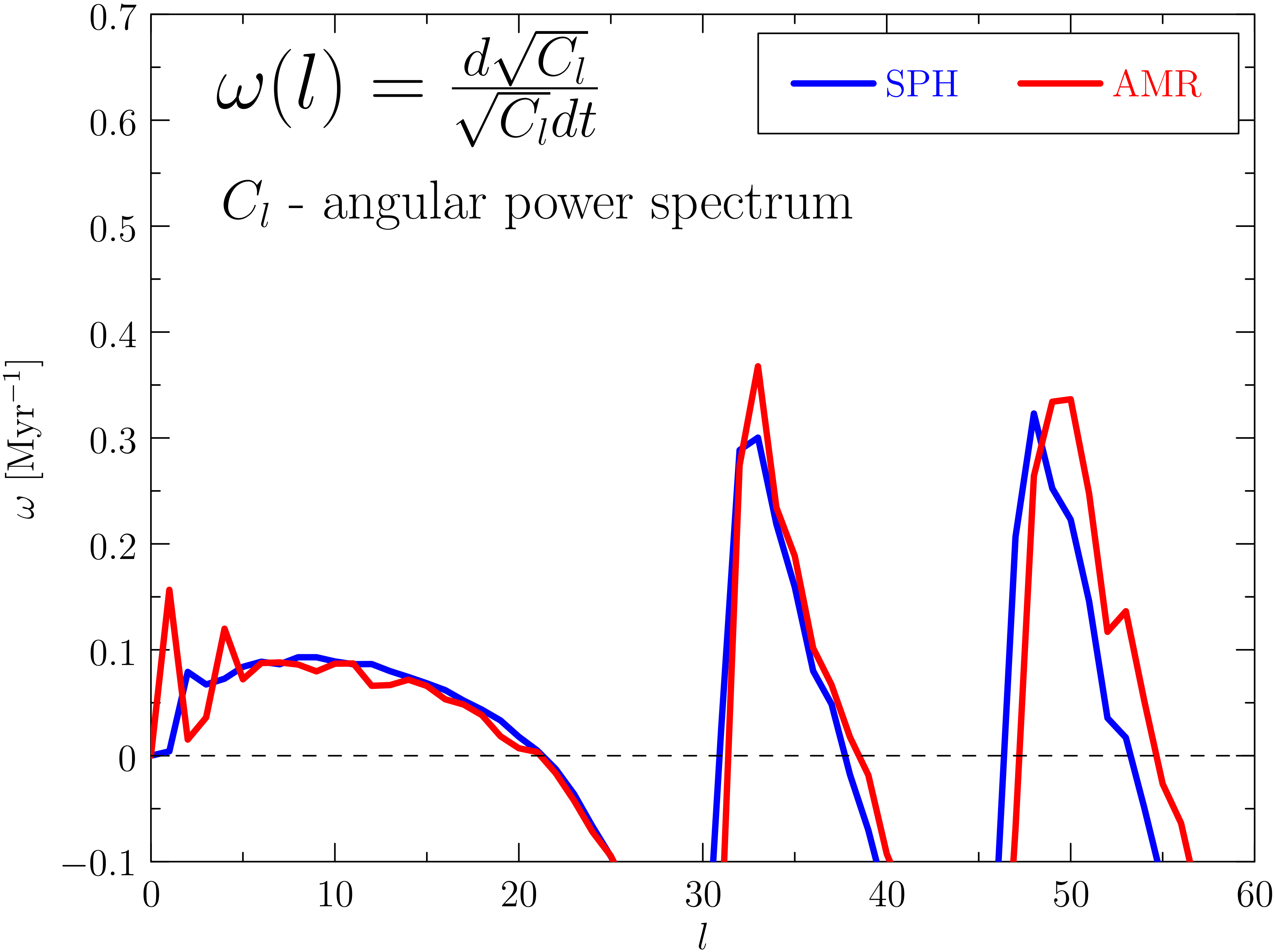
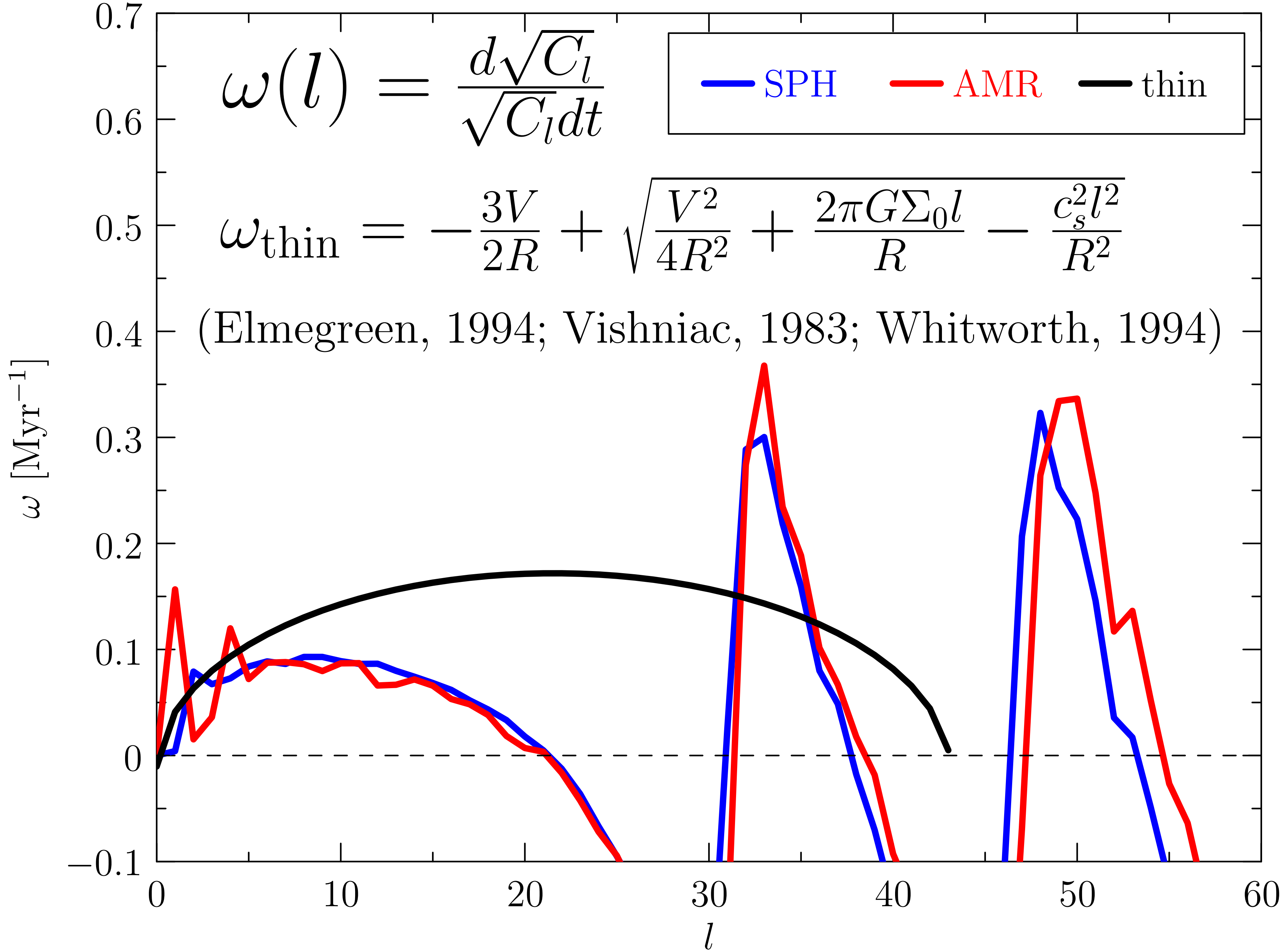


$\omega(l) = \frac{d\sqrt{C_l}}{\sqrt{C_l}dt}$
C_l - angular power spectrum
SPH
AMR
ω [Myr^{-1}]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
0
10
20
30
40
50
60
l



$\omega(l) = \frac{d\sqrt{C_l}}{\sqrt{C_l}dt}$
$\omega_{\rm thin} = -\frac{3V}{2R} + \sqrt{\frac{V^2}{4R^2} + \frac{2\pi G \Sigma_0 l}{R} - \frac{c_s^2 l^2}{R^2}}$
(Elmegreen, 1994; Vishniac, 1983; Whitworth, 1994)
SPH
AMR
thin
ω [Myr^{-1}]
l
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
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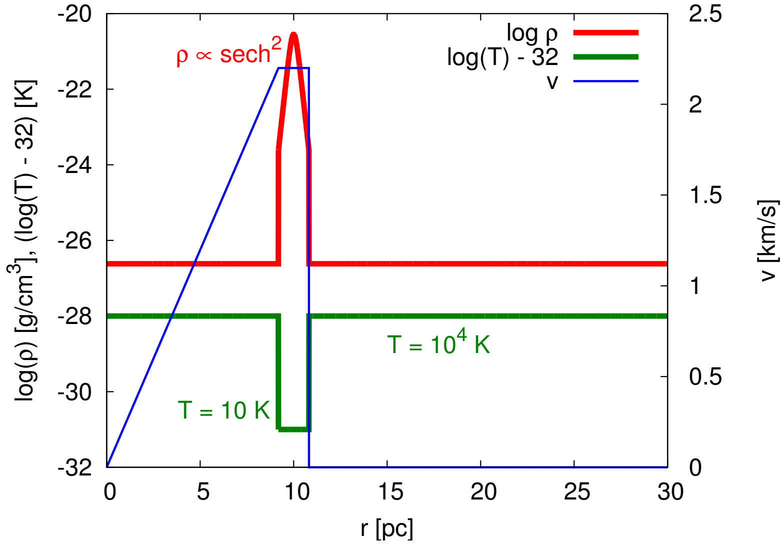
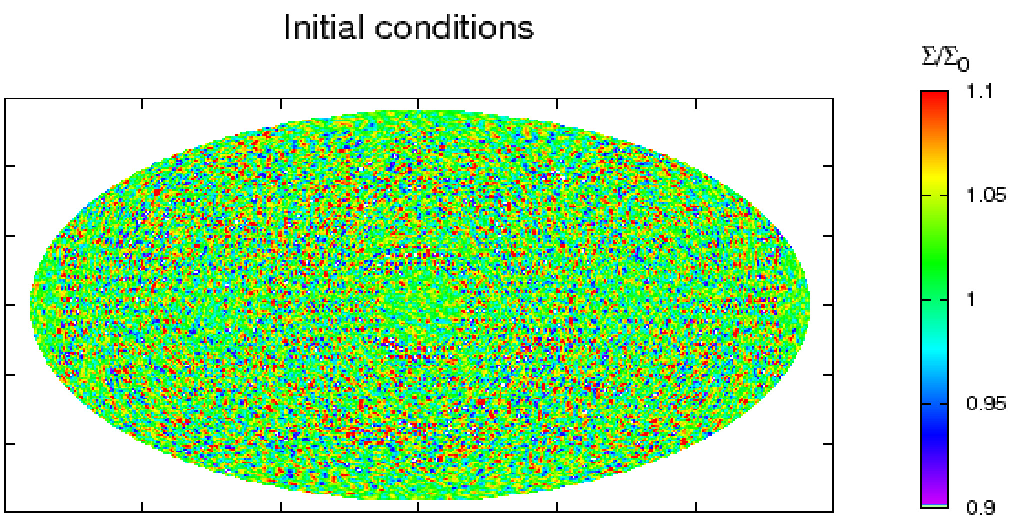
10
20
30
40
50
60

Model setup

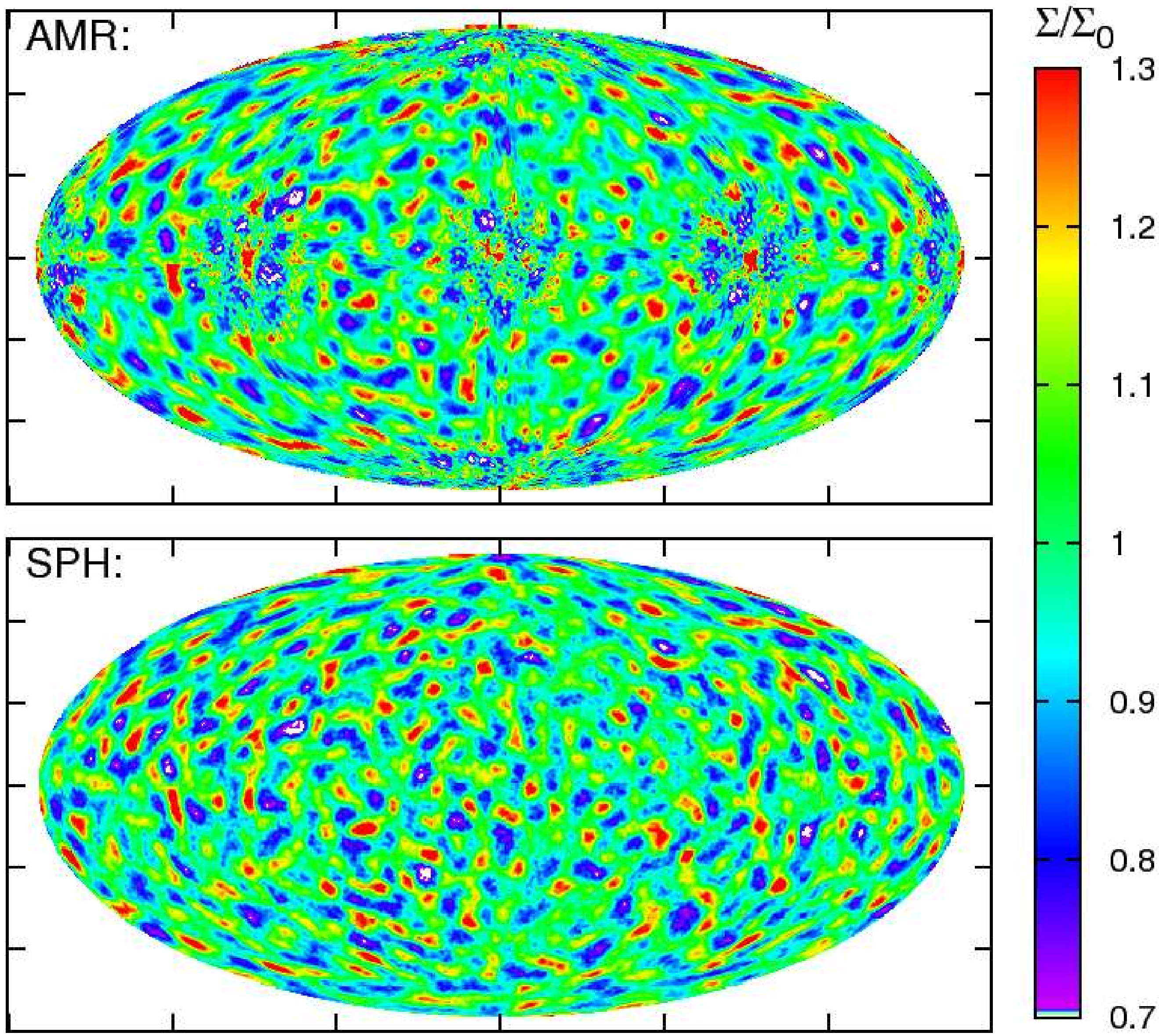
- extremely simplified model to avoid instabilities other than the gravitational one (RT, Vishniac)
- ballistic shell (in a free fall) embedded in a rarefied medium



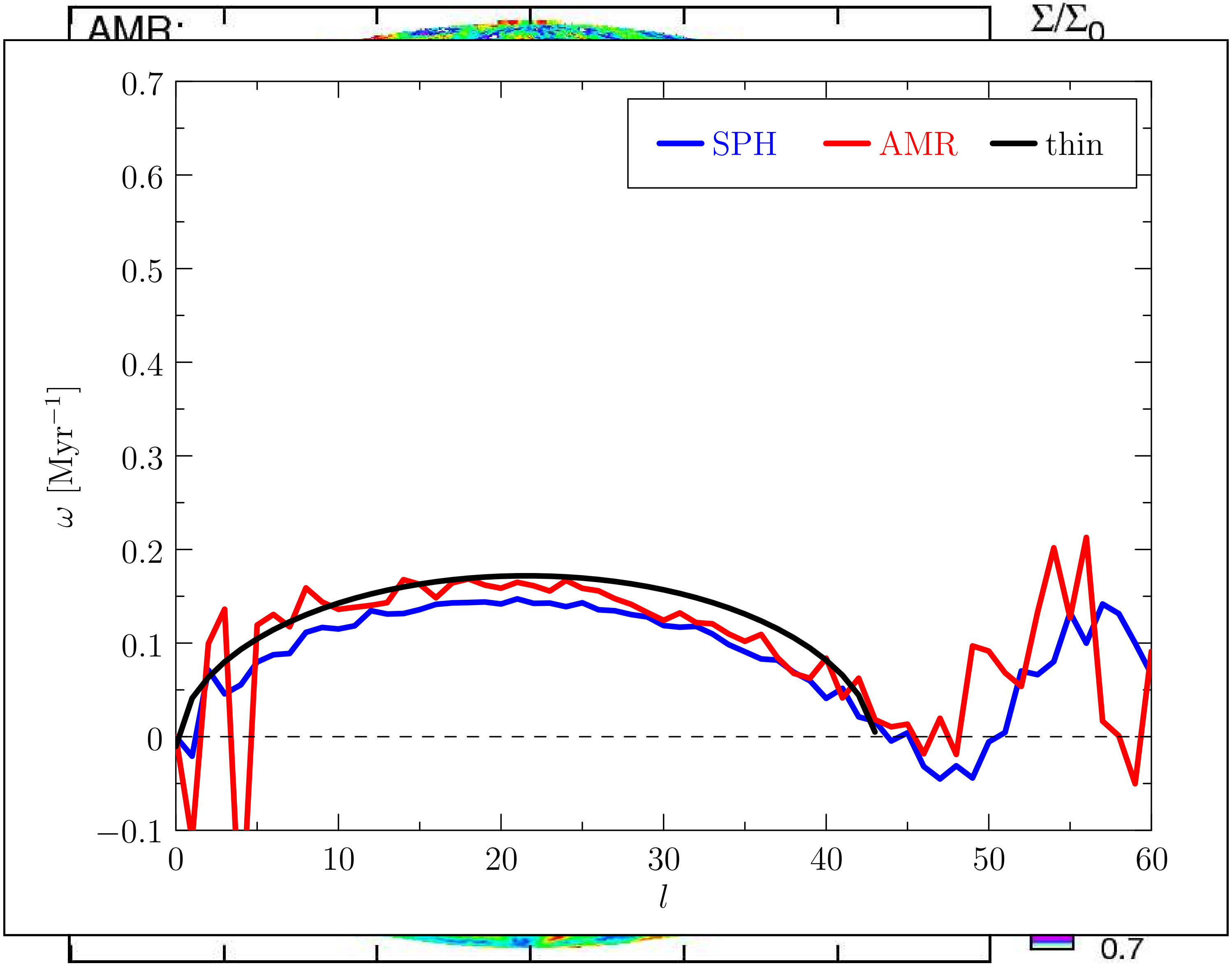


$M_{\mathrm{shell}} = 2 \times 10^4\ \mathrm{M}_\odot$
$T_{\mathrm{shell}} = 10\ \mathrm{K}$
$R_{\mathrm{shell},0} = 10\ \mathrm{pc}$
$V_{\mathrm{shell},0} = 2.2\ \mathrm{km\,s}^{-1}$
$R_{\mathrm{shell,max}} = 23\ \mathrm{pc}$
$P_{\mathrm{ext}} = 10^{-17}$ or $10^{-13}\ \mathrm{dyne\,cm}^{-2}$

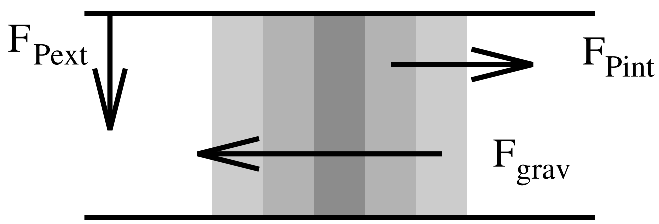
High pressure ambient medium



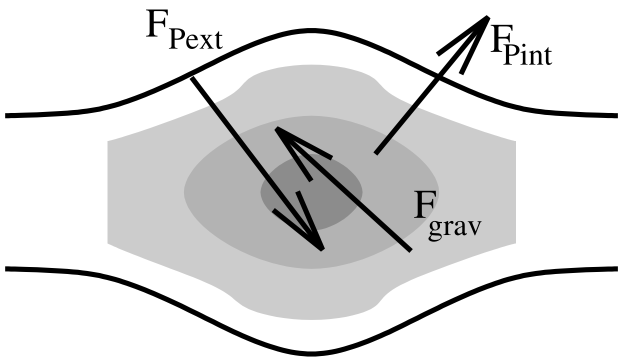
High pressure ambient medium



Dependence of fragment growth rate on $P_{\rm ext}$

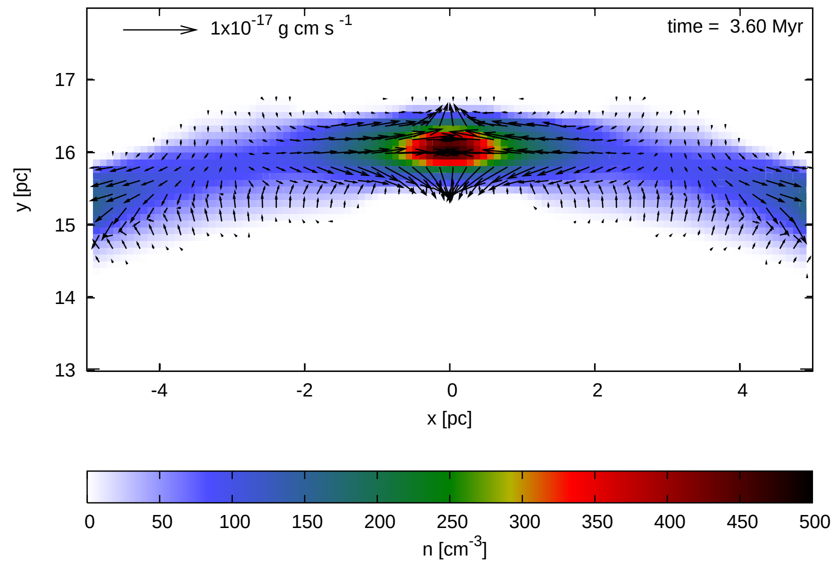


thin shell approx.



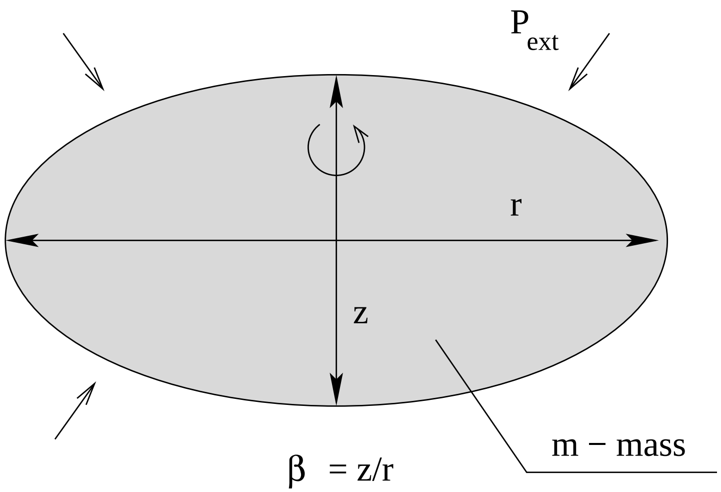
fragments in simulations

- the shell thickness varies across fragments
- external pres. force is not perpendicular to the shell surface



Homogeneous Oblate Rotational Ellipsoid

(Boyd & Whitworth, 2005, A&A, 430, 1059)



Kinetic, gravitational and compressional energy:

$\mathcal{K} = \frac{m}{10}\left(2\dot{r}^2 + \dot{z}^2\right)$

$\mathcal{G} = -\frac{3\,G\,m^2}{5}\,\frac{\cos^{-1}(z/r)}{\left(r^2-z^2\right)^{1/2}}$

$\frac{d\mathcal{B}}{dV} = P_{\rm ext} - P_{\rm int} \;=\; P_{\rm ext} - \frac{m\,c_s^2}{V}$

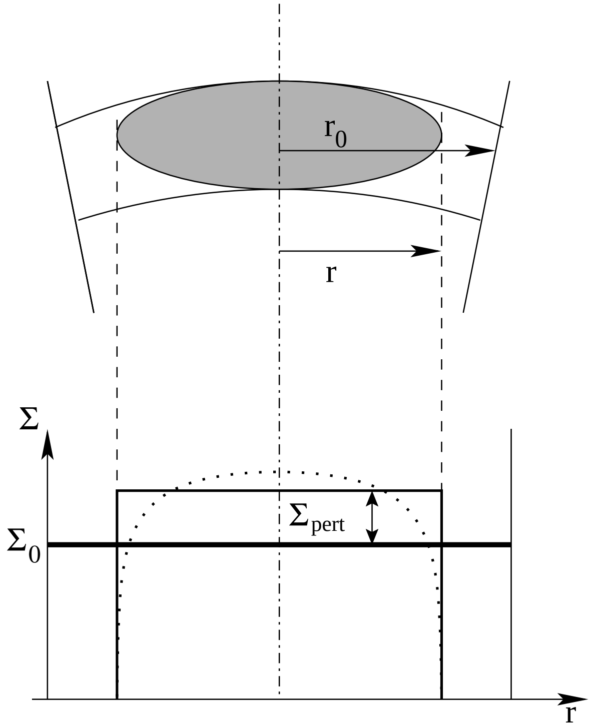
- energy conservation: $\mathcal{E} = \mathcal{K} + \mathcal{G} + \mathcal{B}$
- differentiate with resp. to time:

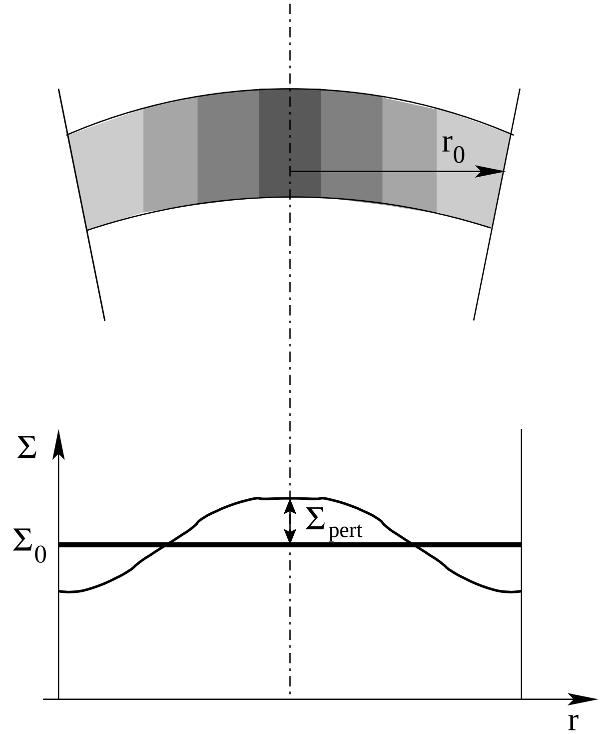
$$\left\{\frac{2\,m\,\ddot{r}}{5} + \frac{d\mathcal{G}}{dr} + \frac{d\mathcal{B}}{dr}\right\}\dot{r} \;+\; \left\{\frac{m\,\ddot{z}}{5} + \frac{d\mathcal{G}}{dz} + \frac{d\mathcal{B}}{dz}\right\}\dot{z} = 0\,.$$

- $\dot{r}$ and $\dot{z}$ independent:

$$\ddot{r} = -\frac{3Gm}{2r^2}\left[\frac{\cos^{-1}\beta}{(1-\beta^2)^{3/2}} - \frac{\beta}{1-\beta^2}\right] - \frac{20\pi P_{\rm ext} r z}{3m} + \frac{5c_s^2}{r}$$

HORE vs. Thin shell





- z given by hydrostatic equilibrium
- non-linear eqs. of motion
- homogeneous ellipsoid

- no vertical structure (inf. thin)
- linearised hydrodyn. eqs.
- sinusoidal perturbations

Perturbation Growth Rate of the mode l

- wavenumber - size of fragment:

$$r_0 = \frac{\pi}{l} R, \qquad \dot{r}_0 = \frac{\pi}{l} V$$

- shell surface density - mass of fragment:

$$m = \pi r^2 \Sigma$$

- fragment collapses with const. acceleration $\ddot{r}$

$$r(t) = r_0 + \dot{r}_0 t + \frac{1}{2} \ddot{r} t^2$$

- shrinks by factor $(1 - \epsilon)$ during time t_ϵ: $r(t_\epsilon) = (1 - \epsilon) r_0$
- perturbation growth rate:

$$\omega_\epsilon = 1/t_\epsilon = -\frac{\dot{r}_0}{2 r_0 \epsilon} + \left(\frac{\dot{r}_0^2}{4 r_0^2 \epsilon^2} - \frac{\ddot{r}}{2 r_0 \epsilon} \right)^{1/2} \qquad (1)$$

- factor ϵ unknown; can be determined by comparison with $\omega_{\rm thin}$
- only range of unstable wavenumbers and relative growth rates

Dispersion relation of the thick shell

$$\omega_\epsilon = \underbrace{-\frac{V}{2R\epsilon} + \Bigg\{\frac{V^2}{4R^2\epsilon^2}}_{\text{stretching}} + \underbrace{\frac{3G\Sigma_0 l}{4R\epsilon}\left[\frac{\cos^{-1}\beta}{(1-\beta^2)^{3/2}} - \frac{\beta}{1-\beta^2}\right]}_{\text{gravity}}$$

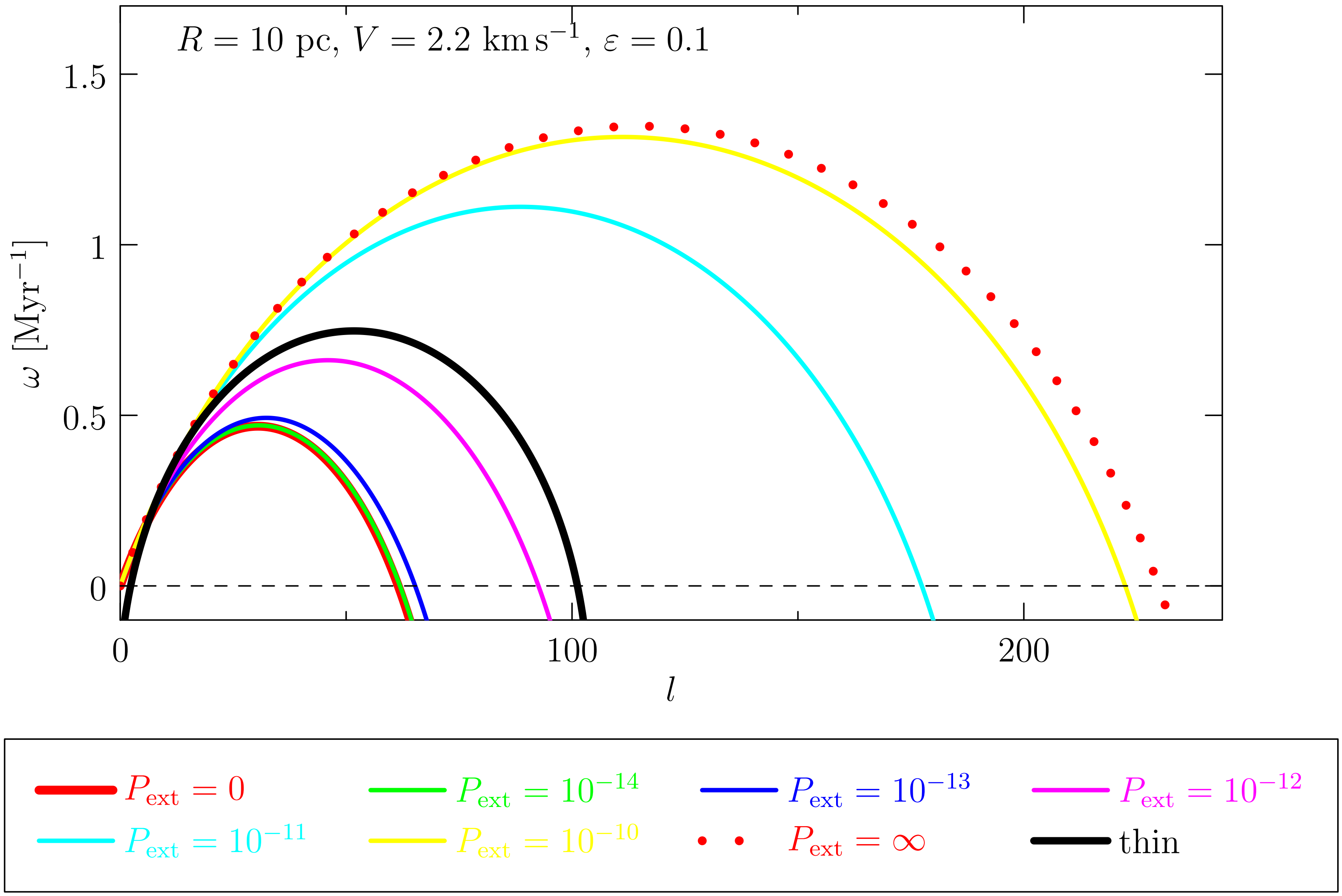
$$+ \underbrace{\frac{1 - P_{\text{ext}} c_s^2 l^2}{3\pi^2 R^2 \epsilon (2P_{\text{ext}} + \pi G \Sigma_0^2)}}_{\text{external pres.}} \underbrace{- \frac{5c_s^2 l^2}{2\pi^2 R^2 \epsilon}}_{\text{internal pres.}} \Bigg\}^{1/2} .$$

Conf. to thin shell:

$$\omega_{\text{thin}} = \underbrace{-\frac{3V}{2R} + \left(\frac{V^2}{4R^2}\right.}_{\text{stretching}} + \underbrace{\frac{2\pi G\Sigma_0 l}{R}}_{\text{gravity}} \underbrace{-\frac{c_s^2 l^2}{R^2}}_{\text{internal pres.}} \left.\right)^{1/2}$$

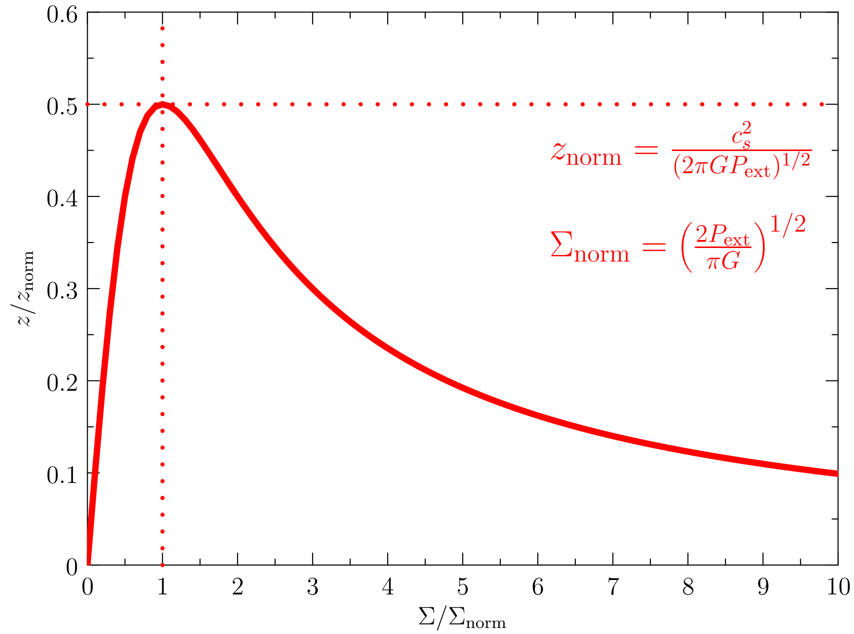
- geometry factor at gravity term
- external pressure term
- dependence on ϵ - shrink fraction

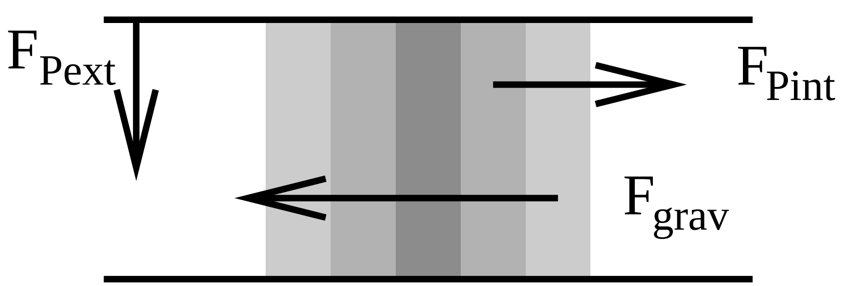
Dispersion relation of the thick shell



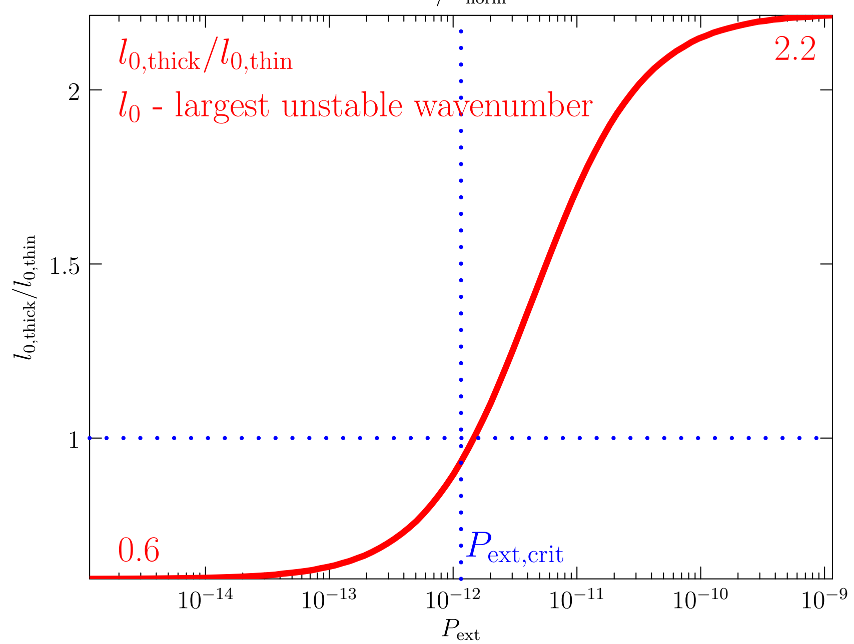
Critical external pressure: $\omega_{\rm thick} \sim \omega_{\rm thin}$

- $\omega_{\rm thick} \sim \omega_{\rm thin}$ for $P_{\rm ext,crit}$ for which $\frac{dz}{d\Sigma} = 0$
  (the shell thickness does not depend on the surface density)

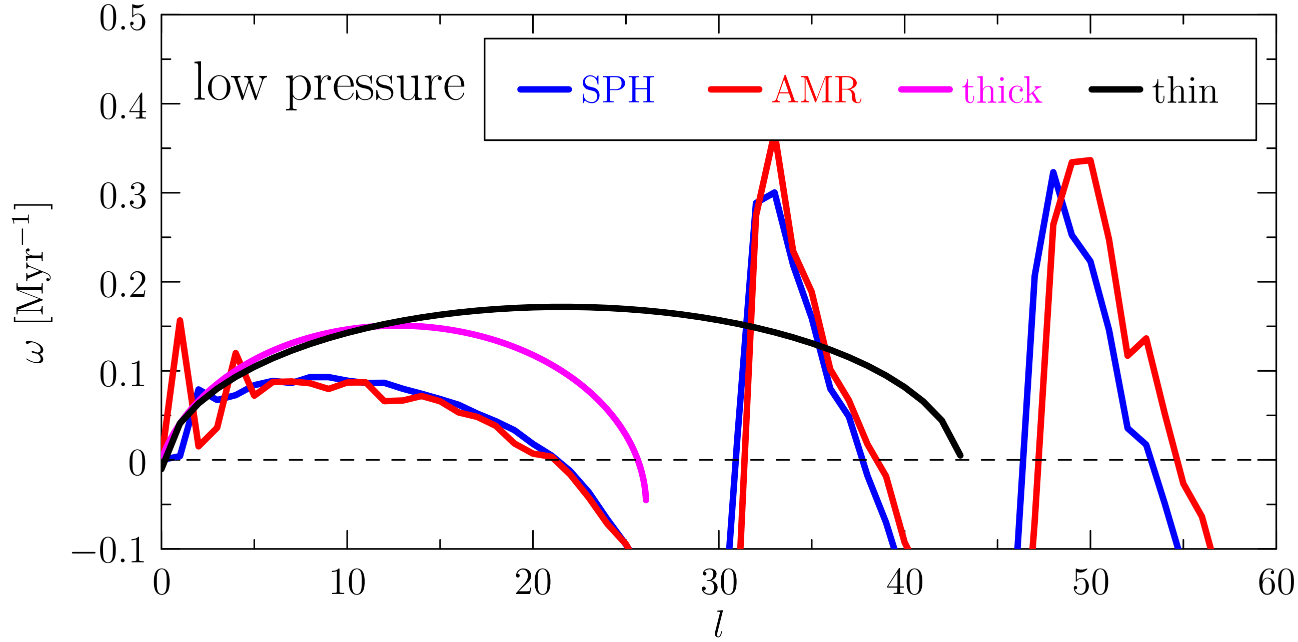


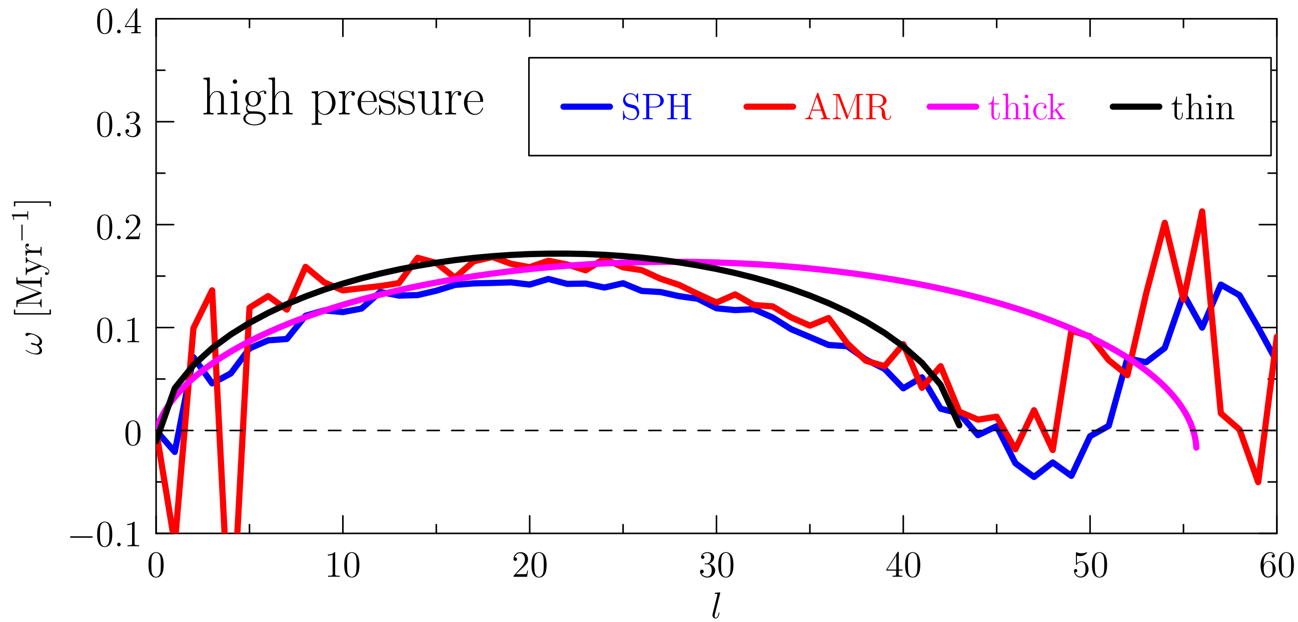


thin shell approx.



New dispersion relation vs. simulations





Discussion

- ω only amplifies initial spectrum (given probably mainly by the turbulence)
- we assume shell confined by thermal pressure from both sides, but instability can be different if the ram pressure confines the shell from outside (accreting shell)
- Vishniac and RT instabilities may have an impact
- would be very nice if we have simulations with $l_0 > l_{0,\mathrm{thin}}$ (very high P_{ext}), but difficult, because very high resolution is needed
- astrophysical consequences: top-heavy IMF (deficit of low mass fragments) in low pres environments
  observational proposal: we plan to use APEX to determine fragment mass function of the Carina Flare supershell (450 pc above the Galactic plane)

Conclusions

- excellent agreement between AMR and SPH, but disagreement with the thin shell approximation
- new dispersion relation (fragment growth rate) for the thick shell embedded in the medium with non-zero pressure
- the new dispersion relation depends on the external pressure, predicts range of unstable wavenumbers different than the one given by the thin shell approximation by factor of 0.6 and 2.2 for $P_{\rm ext} = 0$ and ∞, respectively
- the thick shell dispersion relation is similar to the thin shell one for the pressure for which the shell thickness locally does not depend on its surface density (maximum shell thickness)