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Expanding shells: instability with non-linear terms

Richard Wünsch, Jan Palouš

Astronomical Institute, Academy of Sciences of the Czech Republic ,
Boční II 1401, 141 31 Praha 4, Czech Republic

Abstract. A model of the thin shell expanding into a uniform ambient medium is developed. Density perturbations are described using equations with linear and quadratic terms and the linear and the nonlinear solutions are compared. The interaction of modes on the shell surface is discussed defining the time when the fragments are formed.

1. Introduction

The influence of non-linear terms to the fragmentation of expanding HI shells is discussed extending the linear analysis performed by Elmegreen (1994). We use the similar approach adopted by Fuchs (1996) who described the fragmentation of uniformly rotating self-gravitating disks. If some conditions are fulfilled, the expanding shell may become gravitationally unstable and break to fragments. Inclusion of higher order terms helps to determine when is the shell fragmented with better accuracy than the linear analysis the time.

2. Model used

We consider the cold and thin shell of radius R surrounding the hot interior and expanding with velocity V into a uniform medium of density ρ_0. The intrinsic surface density of the shell Σ is composed of unperturbed part Σ_0 plus the perturbation Σ_1 ($\Sigma = \Sigma_0 + \Sigma_1$). Perturbation Σ_1 results from the flows on the surface of the shell redistributing the accumulated mass.

We consider angular coordinates $\vec{\Theta} = \vec{x}/R$ and angular velocity $\vec{\Omega} = \vec{v}/R$ to describe the surface of the shell. Hydrodynamical equations are

$$\frac{\partial \Sigma}{\partial t} + 2\Sigma\frac{V}{R} + \Sigma R \nabla \cdot \vec{\Omega} + R\vec{\Omega} \cdot \nabla \Sigma = 0 \ , \qquad (1)$$

$$R\frac{\partial \vec{\Omega}}{\partial t} + R^2 \vec{\Omega} \cdot \nabla \vec{\Omega} + V\vec{\Omega} + 3V\vec{\Omega}\frac{\Sigma_0}{\Sigma} = -\frac{c^2}{\Sigma}\nabla\Sigma - \nabla\Phi \qquad (2)$$

where c is constant isothermal sound speed, Φ is gravitational potential of the shell. The second term in the continuity equation and the third and fourth terms in the equation of motion comes from stretching of the region and from the accretion of mass. The Poisson equation is:

$$\Delta\Phi = 4\pi G \Sigma \delta(z) \ , \qquad (3)$$

where z is a space coordinate perpendicular to the shell surface.

The perturbation of the surface density Σ_1 and of the gravitational potential Φ_1 ($\Phi = \Phi_0 + \Phi_1$) are inserted to these equations and they are Fourier transformed with respect to spatial coordinates. In this way we get the set of equations which describe the evolution of the density and of the velocity field on the surface of the shell.

3. Linear solution

The linearized version of equations (1)-(3) gives for the most unstable mode the solution also described by Elmegreen (1994):

$$\omega^{(1,2)}_{\vec{\eta},max} = i\frac{3V}{R} \pm \sqrt{-\frac{V^2}{R^2} - \frac{\pi^2 G^2 \Sigma_0^2}{c^2}} \tag{4}$$

where $\omega^{(1,2)}_{\eta,max}$ is the maximum perturbation growth rate (perturbations of density evolve as $\sim e^{i\omega_\eta t}$). The η_{max} is a dimensionless wavenumber of the most unstable mode of perturbations. If $\omega^{(1,2)}_{\vec{\eta}}$ have got a real part, solution is stable with decreasing oscillations. If not, $\omega^{(1)}_{\vec{\eta}}$ indicates decrease, $\omega^{(2)}_{\vec{\eta}}$ can be imaginary negative and it has meaning of the growth of perturbations, i. e. shell is unstable.

If the shell is unstable, for the certain time, the shell may break to fragments. In Ehlerová et al. (1997) the *fragmentation integral* is defined as:

$$I_{frag}(t) \equiv \int_{t_b}^{t} \omega^{(2)}_{\eta,max}(t')dt' \ , \tag{5}$$

where t_b is the time when the instability begins. The fragmentation time t_f (the time when the shell is decomposed to fragments) is defined as the time when the fragmentation integral is equal to one:

$$I_{frag}(t_f) = \int_{t_b}^{t_f} \omega^{(2)}_{\eta,max}(t')dt' = 1 \ . \tag{6}$$

4. Non-linear analysis

The equations with nonlinear terms are solved by the similar procedure as adopted by Fuchs (1996). We get the set of three ordinary differential equations of the first order describing the evolution of the shell surface density as well as the evolution of the velocity field on the shell surface. The nonlinear terms describe the interaction of the three most unstable modes with wavevectors (of the value η_{max}) inclined at an angle 60^o.

The evolution of the maximum perturbation of the surface density is shown in the left panel of Fig (1). The evolution of Σ_1 can be used to define the fragmentation time of the shell. At advanced stages of the fragmentation the value of the maximum perturbation rises steeply with the time and at some time the value of Σ_1 reaches Σ_0. Consequently we define the fragmentation time t_{frag} as the time when $\Sigma_1(t_{frag}) = \Sigma_0(t_{frag})$.

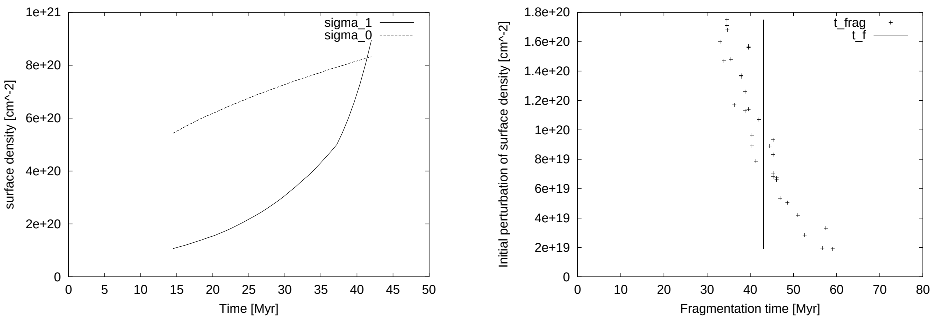

Figure 1. Left panel: The time evolution of Σ_0 and Σ_1. Right panel: Dependence of t_{frag} on a value of the initial perturbation of the surface density. The solid line gives the time defined by the fragmentation integral (6).

In the right panel of Fig (1) the dependence of t_{frag} obtained from the solution of the non-linear equations on the value of the initial perturbation is shown. t_{frag} may be compared to the fragmentation time t_f obtained from the linear analysis defined with the fragmentation integral (6).

5. Conclusions

An interaction of three modes of the type discussed by Fuchs (1996) occurs on the shell surface. The time when fragments form depends on initial conditions. If the initial perturbation of surface density is of the order $\sim 10^{19} - 10^{20}$ cm^{-2}, which is a typical value of inhomogeneities in the ISM, the average value of fragmentation time t_{frag} is approximately of the same value as the time t_f when the fragmentation integral (6) is equal to 1, with a spread of ± 10 Myr depending on the value of the initial perturbation.

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References

Ehlerová, S., Palouš, J., Theis, Ch., Hensler, G. 1997, A&A, 328, 121

Elmegreen, B. G. 1994, ApJ, 427, 384

Fuchs, B. 1996, MNRAS, 278, 985