EXPANDING SHELLS IN LOW AND HIGH DENSITY ENVIRONMENTS

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Abstract. The gravitational instability of expanding shells evolving in a homogeneous and static medium is discussed. In the low density environment ($n = 1$ cm^{-3}), the fragmentation starts in shells with diameters of a few 100 pc and fragment masses are in the range of $5 \times 10^3 - 10^6 M_\odot$. In the high density environment ($n = 10^5 - 10^7$ cm^{-3}), shells fragment at diameters of $\sim$ 1 pc producing clumps of stellar masses. The mass spectrum in both environments is approximated by a power law $dN/dm \sim m^{-2.3}$. This is close to the slope of the stellar IMF. To reproduce the observed mass spectrum of clouds (the spectral index close to ~ -2.0) we have to assume, that the cloud formation time is independent of the cloud size, similarly to the Jeans unstable medium.

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1. The Observed IMF of Clouds and Stars

The mass spectrum of 168 molecular clouds identified by NANTEN in the LMC (Fukui et al., 2001) with radii from 28 to 120 pc and masses m between $4 \times 10^4 M_\odot$ and $3 \times 10^6 M_\odot$ is a power law $dN/dm \propto m^{-\alpha}$ with $\alpha = 1.9 \pm 0.1$. This is steeper than the mass spectrum of clouds in the Milky Way: a large set of different observational results finds a mass spectrum slope $\alpha = 1.3 - 2.0$ over a large range of masses (Combes, 1991; Blitz, 1993).

For the stellar IMF, no convincing observational evidence of place to place variations have been found, but the uncertainties in the slope are rather large (Salpeter, 1955; Kroupa et al., 1993; Scalo, 1998). A typical slope for masses $m \geq 0.5$ M$_\odot$ is $\alpha = 2.3$.

2. The Jeans Instability

The growth rate ω of the gravitational Jeans instability in a homogeneous medium of density ρ is given by the dispersion relation

$$\omega(\lambda) = 2\sqrt{-\frac{c^2\pi^2}{\lambda^2} + \pi\, G\rho}, \tag{1}$$

where λ is the wavelength of the perturbation, c is the sound speed in the medium and G is the constant of gravity. The instability sets in if the right hand side of eq. (1) has a real value.

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The number of fragments of the wavelength λ formed per unit of time within the volume of the radius R is

$$N = \omega \times \frac{R^3}{(\lambda/4)^3}, \tag{2}$$

which implies the mass spectrum $\xi_{Jeans}(m) \equiv \frac{dN}{dm}$

$$\xi_{Jeans}(m) = -\frac{16}{9} R^3 \rho m^{-2} \left[-c^2 \left(\frac{\pi^4 \rho}{6m} \right)^{2/3} + 4\pi G \rho \right]^{1/2}. \tag{3}$$

The high mass end of the Jeans mass spectrum has the power-law slope $\alpha = 2$, it flattens towards the lower masses, since the two terms in brackets have opposite signs, and terminates at the Jeans mass $m_{Jeans} = \frac{\pi^{5/2}}{48} G^{-3/2} c^3 \rho^{-1/2}$ (see also Palouš et al., 2002).

3. The Fragmentation of an Expanding Shell

The dispersion relation of a shell expanding into a homogeneous medium has been derived by Elmegreen (1994) and Wünsch and Palouš (2001):

$$\omega(\eta) = -\frac{3V_{sh}}{R_{sh}} + \sqrt{\frac{V_{sh}^2}{R_{sh}^2} - \frac{c_{sh}^2 \eta^2}{R_{sh}^2} + \frac{2\pi G \Sigma_{sh} \eta}{R_{sh}}}, \tag{4}$$

where R_{sh} is the radius of the shell, V_{sh} is its expansion speed, Σ_{sh} is its column density, c_{sh} is the speed of sound within the wall and $\eta = 2\pi R_{sh}/\lambda$ is the dimensionless wavenumber.

The instability begins at the time t_b, when $\omega(\eta) > 0$ for some $\eta = \eta_{fast}$ for the first time. We define a fragmentation integral

$$I(\eta, t) = \int_{t_b}^{t} \omega(\eta, t')dt', \tag{5}$$

which represents the evolution of ω for a given value of η. The mass spectrum can be constructed analogically to the previous case, but we use the fragmentation integral $I(\eta, t)$ instead of $\omega(\lambda)$.

Neglecting the external pressure and assuming the continuous energy input L, the self-similar solution of the thin expanding shell gives $R_{sh}(t)$, $V_{sh}(t)$ and $\Sigma_{sh}(t)$ (Castor et al., 1975), and the time t_b can be computed. In the low density medium, $n = 10^{-1}$ cm^{-3} with $c_{sh} = 1.0$ km s^{-1} we get $t_b = 91.1$ Myr; in the high density medium, $n = 10^7$ cm^{-3} with $c_{sh} = 0.3$ km s^{-1} it is $t_b = 0.01$ Myr. The mass spectrum of fragments at $t = 10 \times t_b$ is shown in the left panel of Fig. 1. The slope of the mass spectra is always close to $\alpha = 2.3$, the mass range of fragments

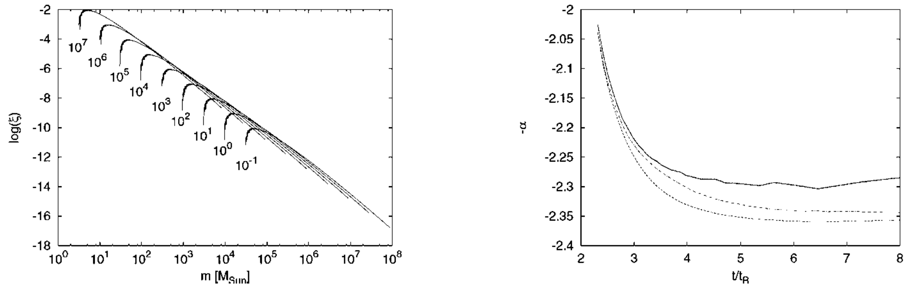

Figure 1. Left panel: The mass spectrum of fragments of an expanding shell with $L = 10^{51}$ erg/Myr, $c_{sh} = 1$ km/s. The density n of the ambient medium varies from 10^{-1} to 10^7 cm^{-3}. Right panel: The time evolution of the spectral index α. Solid line: $L = 10^{51}$ erg/Myr, $c_{sh} = 1.0$ km/s, $n = 1$ cm^{-3}; dotted line: $L = 10^{51}$ erg/Myr, $c_{sh} = 1.0$ km/s, $n = 10^7$ cm^{-3}; dash-dotted line: $L = 10^{53}$ erg/Myr, $c_{sh} = 1.0$ km/s, $n = 1$ cm^{-3}; dashed line: $L = 10^{51}$ erg/Myr, $c_{sh} = 0.3$ km/s, $n_0 = 1$ cm^{-3}.

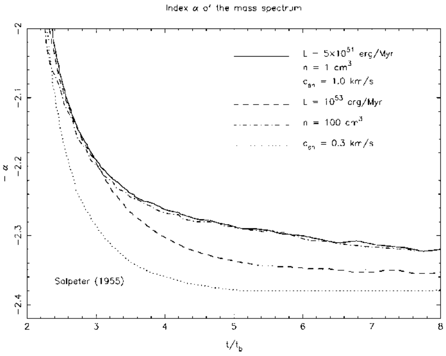

Figure 2. The time evolution of the spectral index α from numerical simulations. The basic model (solid line) has parameters $L = 5 \times 10^{51}$ erg/Myr, $c_{sh} = 1.0$ km/s, $n = 1$ cm^{-3}. Other models deviate from the basic one in indicated quantities. The horizontal line at $\alpha = 2.35$ shows the slope of the IMF proposed by Salpeter (1955).

shifts for increasing values of $n = \rho/\mu$ (we use $\mu = 1.3\mu$) to lower masses. Stellar masses are achieved for $n = 10^5 - 10^7$ cm^{-3}. The time evolution of the spectral mass index α is shown in the right panel of Figure 1. Independently of L, c_{sh} and n its value starts at 2, it is growing with time and after $t \sim 4 \times t_b$ it stays nearly constant between 2.3–2.35.

The thin shell approximation has been adopted in numerical simulations, which include the radiative cooling and pressure of the ambient medium. The turbulent nature of the ISM is considered with the inclusion of density and velocity gradients. The computer code used in this paper is described in Ehlerová and Palouš (2002) and Elmegreen et al. (2002). The mass spectrum is shown in Figure 2: it is similar to the self-similar solution. For $t > 4 \times t_b$ it has the slope $\alpha \in (2.32, 2.38)$.

4. Discussion, Conclusions and Future Investigation

The gravitational instability of thin and expanding shells produces the mass spectrum with the power-law index $\alpha = 2.3$. This is rather close to the slope of the Salpeter IMF for stars. The observed mass spectrum of clouds is more shallow corresponding to the mass spectrum produced by the 3-dimensional Jeans instability, which has the spectral index $\alpha = 2.0$. To get flatter slopes, we speculate that some process of merging between clouds is needed, which would increase the number of massive clouds at the expense of the low mass clouds. Conclusions are based on the self-similar solution and on numerical simulations using the thin shell approximation. In the future, we shall explore mass spectra produced in different galaxy models (Palouš et al. 2003) to check how universal the above conclusion is.

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