Hydrodynamic Simulations of Expanding Shells: The Gravitational Instability

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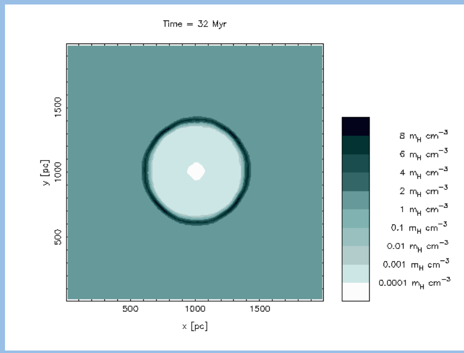
Abstract: We are working on the 3D numerical model of the expanding self-gravitating shell using the ZEUS hydrocode. Simulations should verify previous studies of gravitational instability of the expanding shell using the thin layer approximation.

Linearized equations of this problem were solved by Elmegreen (1994) who obtained the criteria for the shell instability. The energy is inserted into a small volume in the static and homogeneous medium forming an expanding spherical shock. After the initial period of fast expansion the temperature in the shock decreases due to the radiative cooling and the shock collapses into the thin shell. In the initial period of the evolution the shell is gravitationally stable due to squeezing connected to the fast expansion. Later, when it decelerates, the gravity starts to be important. During its subsequent supersonic expansion the accumulation of the ambient medium may trigger gravitational instability and fragmentation.

We are interested in the mass spectrum of the fragments which may be compared with the observed mass spectrum of the giant molecular clouds and the stellar initial mass function.



The density field in the 3D simulation of the expanding shell in the cartesian grid 80x80x80. The energy is inserted continuously with rate 10^{53}erg/15Myr into the homogeneous ISM with the density $1.3 m_H \mathrm{cm}^{-3}$.

Analytical approach: linear analysis of the gravitational instability

The hydrodynamic equations:

$$\frac{\partial \Sigma}{\partial t} + 2\Sigma \frac{V}{R} + \nabla_T \cdot (\Sigma R \Omega) = V\rho 0$$

$$R\frac{\partial \Omega}{\partial t} + V\Omega + 3V\Omega\frac{\Sigma_0}{\Sigma} + R^2(\Omega\cdot\nabla_T)\Omega = -\frac{c^2}{\Sigma}\nabla\Sigma - \nabla\Phi$$

+ the Poisson equation:

$$\Delta\Phi = 4\pi G\Sigma\delta(r-R)$$

→ Linearization, perturbation analysis, Fourier transform, assuming $\Sigma, \Omega \sim \exp(\omega t)$ →

$$\omega(\eta, t)^{(1,2)} = -\frac{3V}{R} \pm \sqrt{\frac{V^2}{R^2} - \frac{\eta^2 c^2}{R^2} + \frac{2\pi G \Sigma_0 \eta}{R}}$$

- perturbation growth rate, instability criteria (Elmegreen, 1994)

Σ, Σ_0 . . . the perturbed an the unperturbed surface density

Ω . . . 2D velocity of the surface flows

R, V . . . the radius and the expansion velocity of the shell

ρ_0 . . . the density of the ambient medium

c . . . the sound speed in the shell

η . . . dimensionless wavenumber, $\eta = kR$, where k is the wavenumber in common sense

∇_T . . . 2D gradient in the surface of the shell

The mass spectrum of fragments:

The number of fragments:

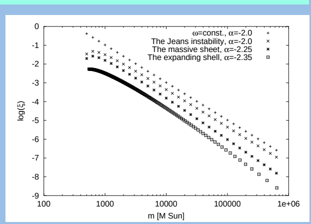
$$N(\eta) \sim I_f \frac{R^2}{\lambda^2}$$

where $I_f(\eta, t) = \int \omega(\eta, t)dt$ is the fragmentation integral, which measure the stage of the fragment evolution, λ is the "wavelength" of the instability

Mass spectrum:

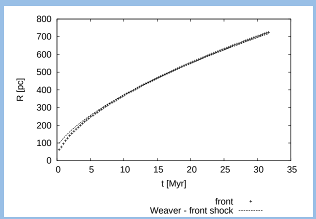
$$\frac{dN}{dm} \sim m^{-\alpha}$$

The mass spectra derived under a different physical conditions. The scale-free instability (growth rate of the fragments does not depend on their mass) and the Jeans collapse yield the spectrum index $\alpha = 2.0$, while the spectra obtained from the accreting mass sheet and the expanding shell are harder ($\alpha = 2.25$ and the "Salpeter value" $\alpha = 2.35$). (Palouš et al., 2002)

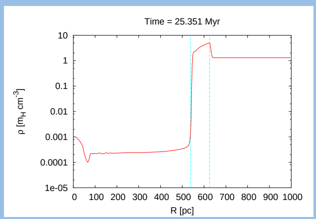


Comparison to Weaver solution:

We compare the 1D (400x1x1) spherically symmetric adiabatic simulation with the continuous energy input (in the form of heat) to the "classical" analytical Weaver solution (Weaver, 1977).



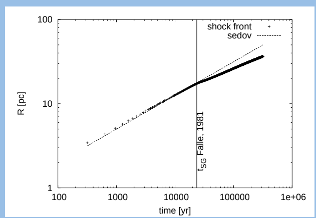
Position of the shock front. In the later stage of the evolution is the discrepancy less than 5 zones, which is the scale on which the artificial viscosity acts.



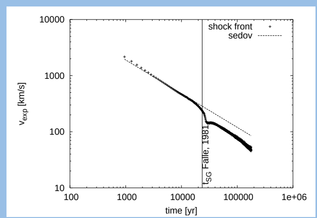
Radial profile of the density in the simulation. Cyan lines are positions of the shock front (R_{sf}) and the contact discontinuity at $0.86R_{sf}$ in the Weaver solution.

Testing of the radiative cooling:

We setup the astrophysical blastwave problem with an abrupt energy input 10^{51}erg, which is described by the Sedov solution in the initial period. At the time t_C the accreted material cools catastrophically and collapses into the thin shell. This process was described by Falle, 1981.

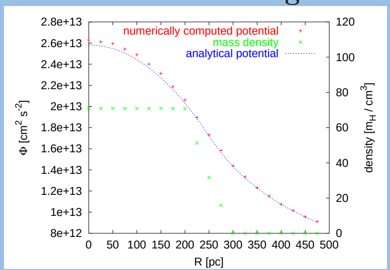


The evolution of the shock radius leaves the Sedov solution at the time when the fast cooling occurs and the shell collapses. This time coincides perfectly with the t_C time, predicted by Falle for this effect.



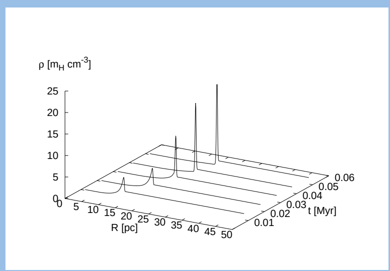
The velocity of the shock decreases suddenly at the time of the shell collapse. It is due to the lack of pressure behind the shock front, because the gas flows to the underpressured cold shell. This effect was also described by Falle.

Poisson solver testing:



We use the patched version of the internal ZEUS Poisson solver, which uses the DADI (Dynamic Alternating Direction Implicit) method. The homogeneous sphere in the centrum of the 20x20x20 zones area is set up as the simplest test of the Poisson solver. The maximum relative error is of the order 10^{-3}.

Radial profiles of the density near the t_C time (0.027 Myr). Before this time they fit almost exactly the adiabatic Sedov solution with the compression factor 4. Then the collapse starts and the compression factor increases several times during a short period.



Conclusions and the next steps:

- the expanding self-gravitating shell may be the way how to obtain the mass spectrum of fragments close to the stellar IMF
- the ZEUS hydrocode and the cooling routine were tested and seem to be applicable for this problem
- the Poisson solver basically works, but it needs still some tuning
- the aim is to verify the analytical results numerically

References:

Elmegreen, B. G., 1994, ApJ, 427, 384

Falle, S. A. E. G., 1981, MNRAS, 195, 1011

Palouš, et al., 2002, MNRAS, in preparation

Weaver et al., 1977, ApJ, 218, 377