Gravitational instabilities of expanding shells

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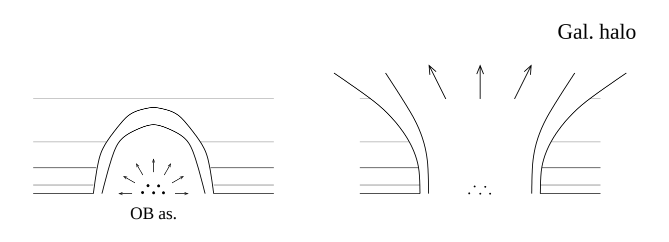
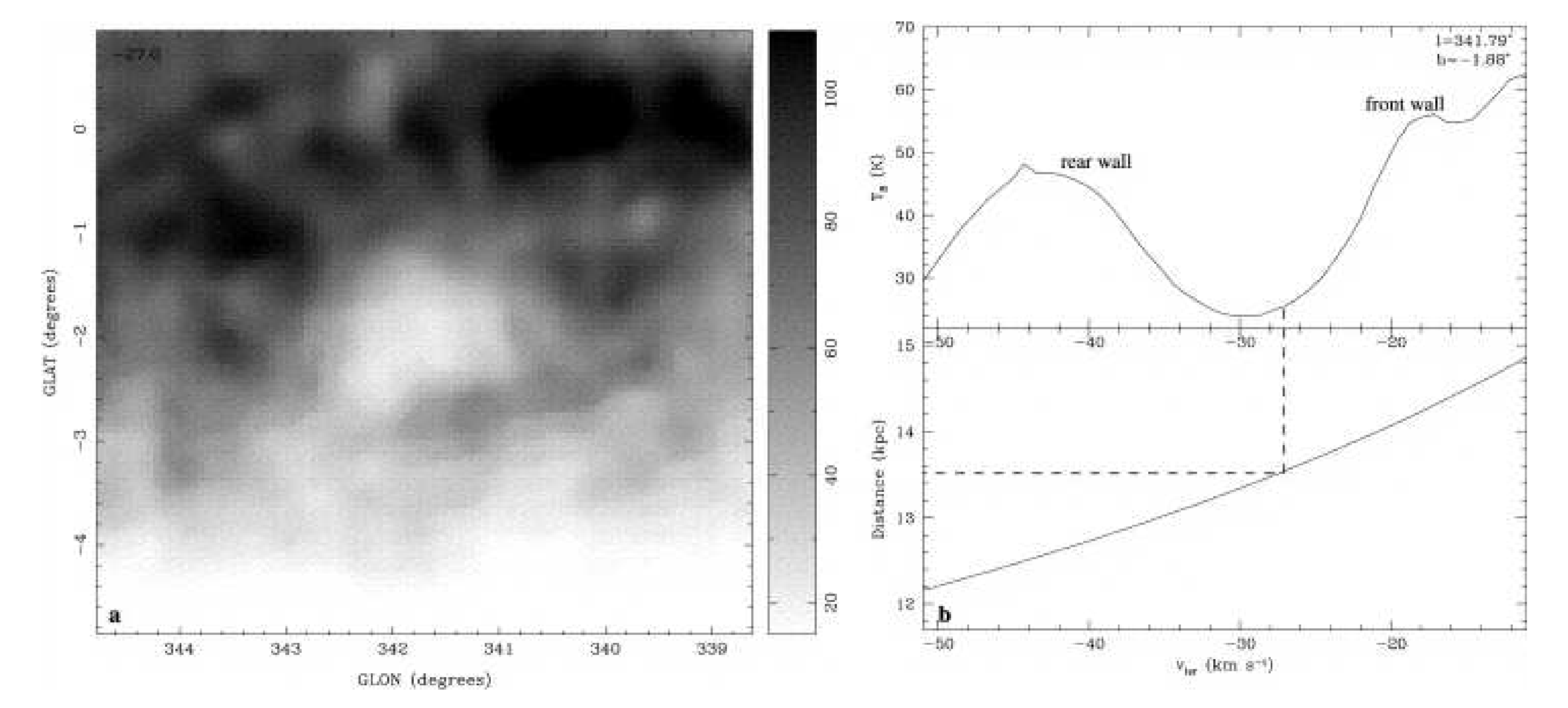
1. HI shells and supershells
2. The gravitational instability and the mass spectrum of fragments
3. Hydrodynamic simulations using the Zeus code
4. The Poisson solver
5. Future plans

HI shells

- **basic characteristics:**
  $R \sim 10\ pc - 2$ kpc, $E \sim 10^{51} - 10^{54}$ erg, $v_{exp} \sim 5 - 30$ km/s
- **Observations:**
  MW: Heiles (1979); recent obs.: McClure-Griffiths et al. (2002)
  external galaxies: LMC (Kim, 1998), SMC (Stanimirovic, 1999)
  M31, M33, M101, HoII, IC10
- **origin - SN explosion, OB association, group of OB as., encounter with HVC or a dwarf galaxy, GRB**
- **Why are HI shells interesting?**
  - **Propagating Star Formation:**
    OB as. $\rightarrow$ exp. HI shell $\rightarrow$ fragments $\rightarrow$ new stars
  - **Disc Halo Connection**



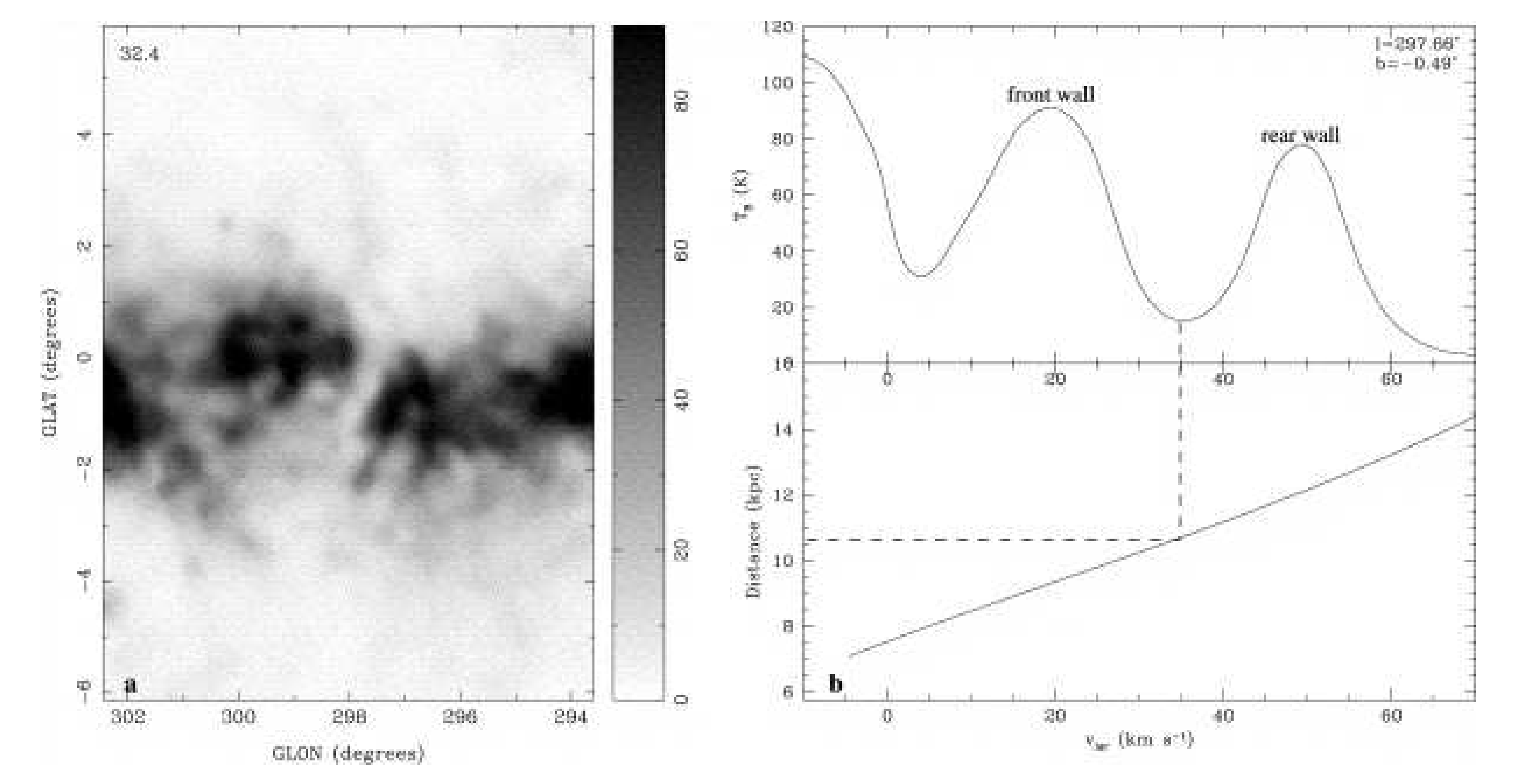
Example of the HI shell



Galactic shell GSH 342-02-27 $\Delta l = 4.7$, $\Delta b = 2.5$

(McClure-Griffiths et al., 2002)

Example of the chimney



Galactic chimney GSH 298-01+35 $\Delta l = 0.8$**,** $\Delta b > 12$
(McClure-Griffiths et al., 2002)

$d = 10.5$ kpc $\rightarrow 75$ pc $\times$ 1 kpc, $E \sim 1.1 \cdot 10^{52}$ erg

Gravitational instability - the linear analysis

Elmegreen (1994)

Assume a cold and thin shell of the radius r expanding with the velocity v into the medium of the density ρ_0. HD equations and the Poisson equation on the shell surface:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \Omega) = v \rho_0,$$

$$\frac{\partial}{\partial t} (\Sigma \Omega) + (\Omega \cdot \nabla) \Sigma \Omega = -\Sigma \nabla \Phi - c^2 \nabla \Sigma$$

$$\Delta \Phi = 4 \pi G \Sigma \delta (R - r)$$

Σ	–	surface density of the shell
Ω	–	2D angular velocity of the surface flows
c	–	sound speed in the shell
Φ	–	gravitational potential

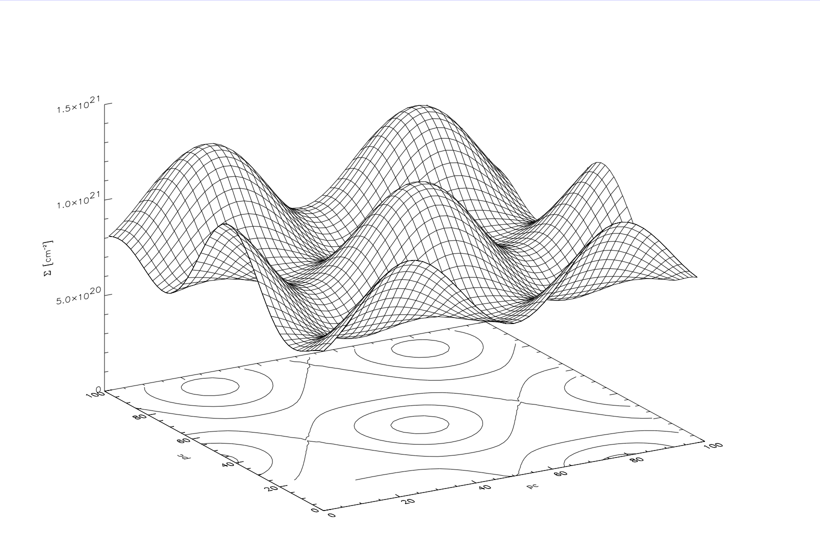
$\rightarrow$ **linearization, stability analysis of the Fourier modes**
(parametrized by the dimensionless wavenumber $\eta = kr$):

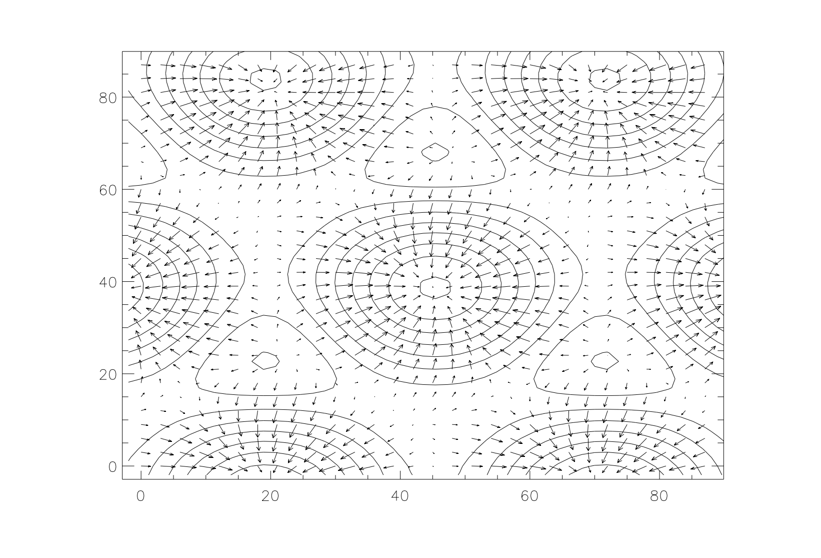
$$\omega(\eta) = -\frac{3v}{r} + \sqrt{\frac{v^2}{r^2} + \frac{2\pi G \Sigma_0 \eta}{r} - \frac{c^2 \eta^2}{r^2}}$$

Gravitational instability - the quadratic terms

Wünsch & Palouš (2001)

- similar approach as Fuchs (1996) – uniformly rotating disk
- quadratic terms couple Fourier modes inclined at an angle 60 deg
- coupling weaker than in the case of the disk → should be tested numerically





up: the surface density which appropriates to the triplet of interacting modes

left: the velocity field of the surface flows of mass in the same case

Mass spectrum of fragments

- can be derived from the dispersion rel. $\omega(\eta)$ (from the linear analysis)
- the fragmentation integral:

$$I_f(\eta, t) = \int_{t_B}^{t} \omega(\eta, t')dt' \ ,$$

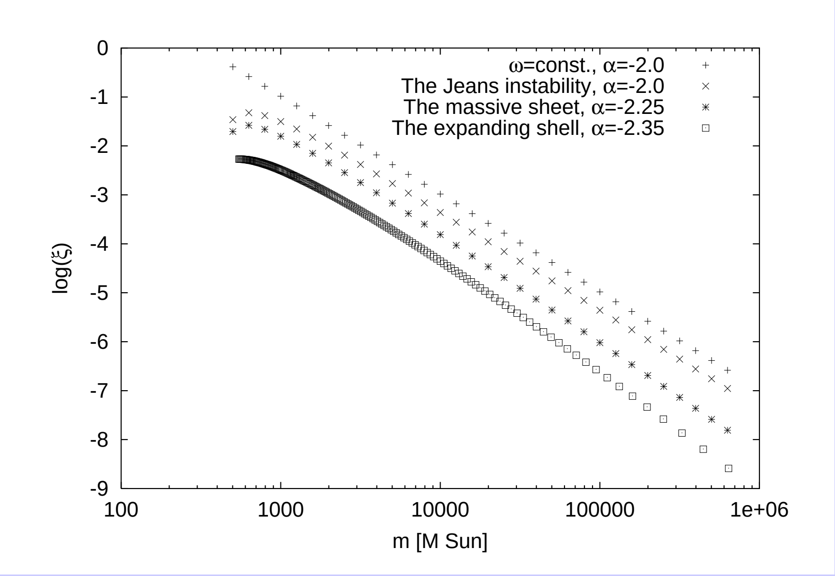
where t_B is the time when the instability begins

- the mass spectrum:

$$dN \sim I_f(\eta, t) \times \frac{r^2}{\lambda^2} d\eta$$
$$\rightarrow dN \sim m^{-\alpha} dm \ ,$$

where α is mass spectrum index

expanding shell yields the mass spectrum index $\alpha = 2.35$ at high mass end (very close to the stellar IMF) independently on the parameters of the shell and on the stage of its evolution, conf. to other physical situations



The ZEUS hydrocode (Stone & Norman, 1992)

- **3D second-order accuracy hydrodynamic code**
- **written in a covariant formalism $\Rightarrow$ works in arbitrary orthogonal coordinates**
- **based on method of finite differences and finite volumes (simple, fast and robust)**
- **exists also in parallel version (but we don't use it yet)**
- **solves the set of equations:**

The hydrodynamic equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \rho \vec{v} = -\nabla p - \rho \nabla \Phi$$

$$\rho \frac{\partial}{\partial t}\left(\frac{e}{\rho}\right) + \rho \vec{v} \cdot \nabla \left(\frac{e}{\rho}\right) = -p \nabla \cdot \vec{v}$$

+ the equation of state:

e.g. $p = (\gamma - 1)e$

ZEUS: What had to be changed

- new Poisson solver (we have two: DADI solver based on the original Zeus method and the new multigrid solver)
- new cooling routine implemented (cooling function was kindly provided by Prof. Rozyczka)
- additional pressure (in reality connected to the mag. field or the turbulence)
- we fix a serious bug in the total energy advection scheme (TEA conserves energy to machine round-off, but negative pressures may occur)
- several smaller bugs were fixed (in collaboration with A. Gawryszcak)

ZEUS: Model of the shell

We insert an energy and a mass into the medium of density $\rho_0 = n_0 \times \mu$ and the temperature T_0

- **initially**

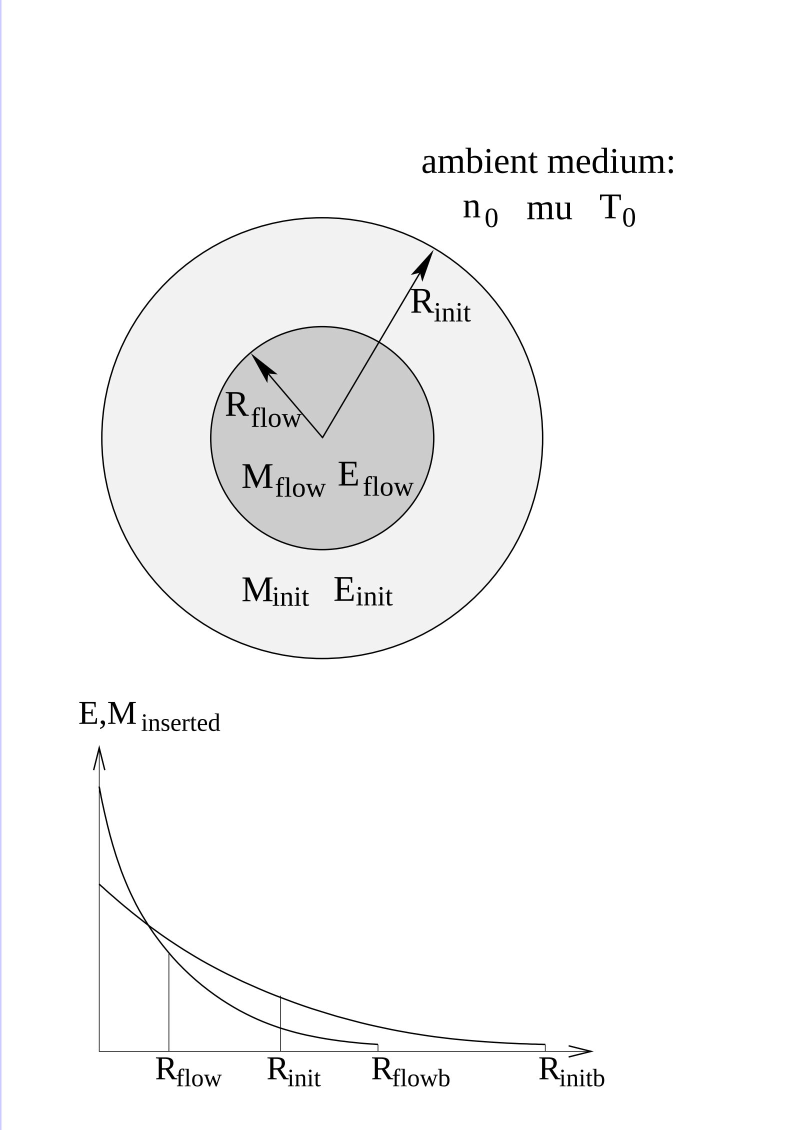
  (**init sphere,** E_{init}, M_{init} , R_{init}, R_{initb})

- **continuously**

  (**flow sphere,** E_{flow}, M_{flow} , R_{flow}, R_{flowb})

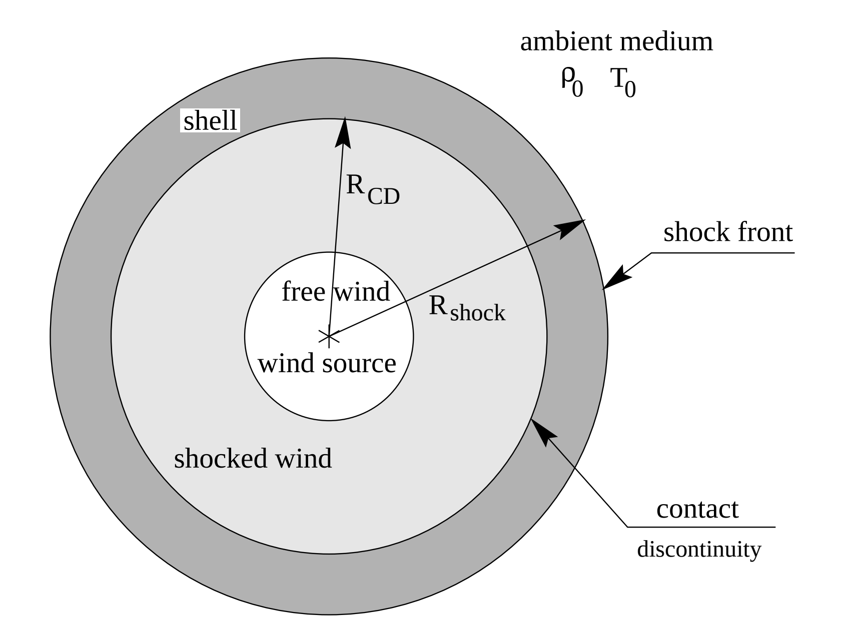
- **or both**

The energy and the mass are inserted with radial distribution $\sim \exp(-r/R_{init,flow})$ to prevent from large gradients.



The Weaver's solution: test of the HD code

- derived by Weaver et al. (1977)
- semi-analytical spherically symmetric solution of HD equations for the problem of stellar wind bubble
- we use the adiabatic version (no cooling)



shock front position:

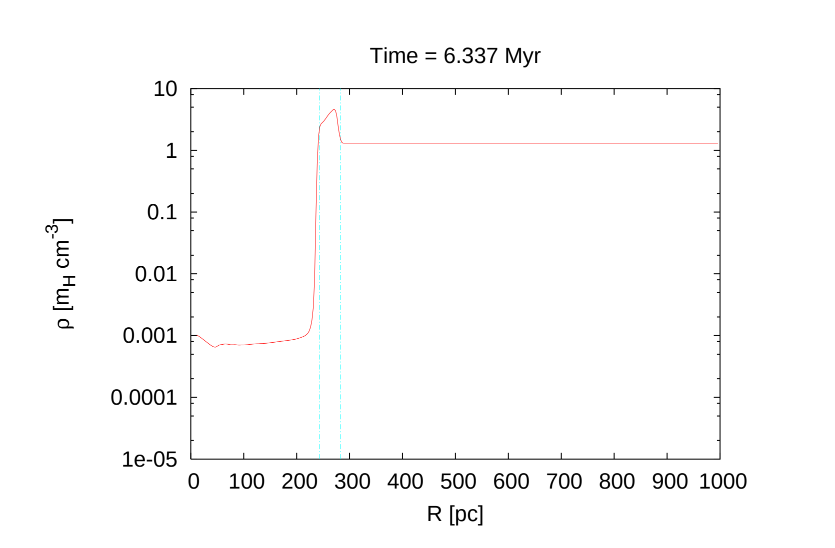
$$R_{\rm shock} = 0.88 \left(\frac{Lt^3}{\rho_0}\right)^{1/5}$$

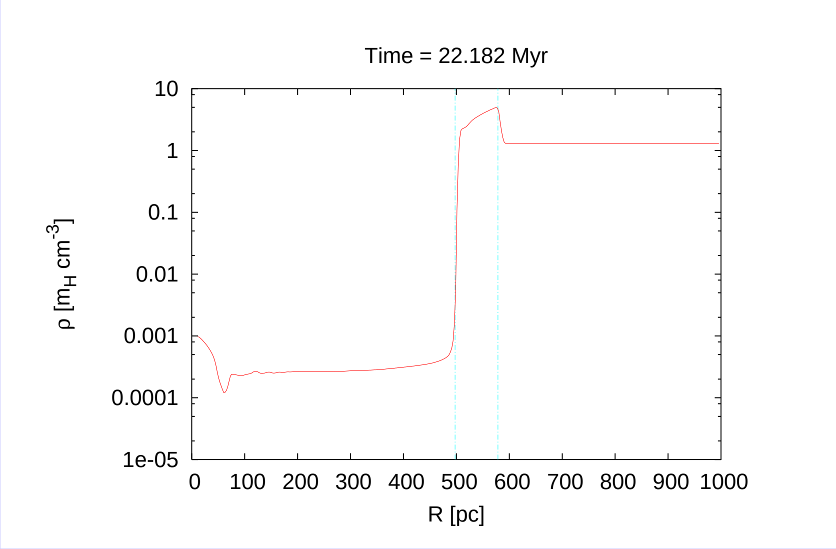
contact discontinuity position:

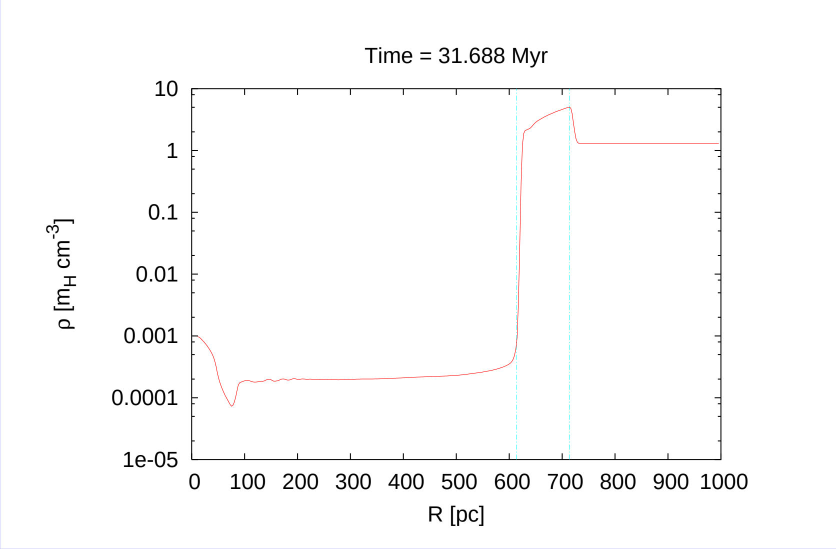
$$R_{\rm CD} = 0.86 R_{\rm shock}$$

Weaver's solution vs. the simulation

- 1D spherically symmetric simulations, 400x1x1 grid
- the shock front and the contact discontinuity remains at the correct postion during the whole computation



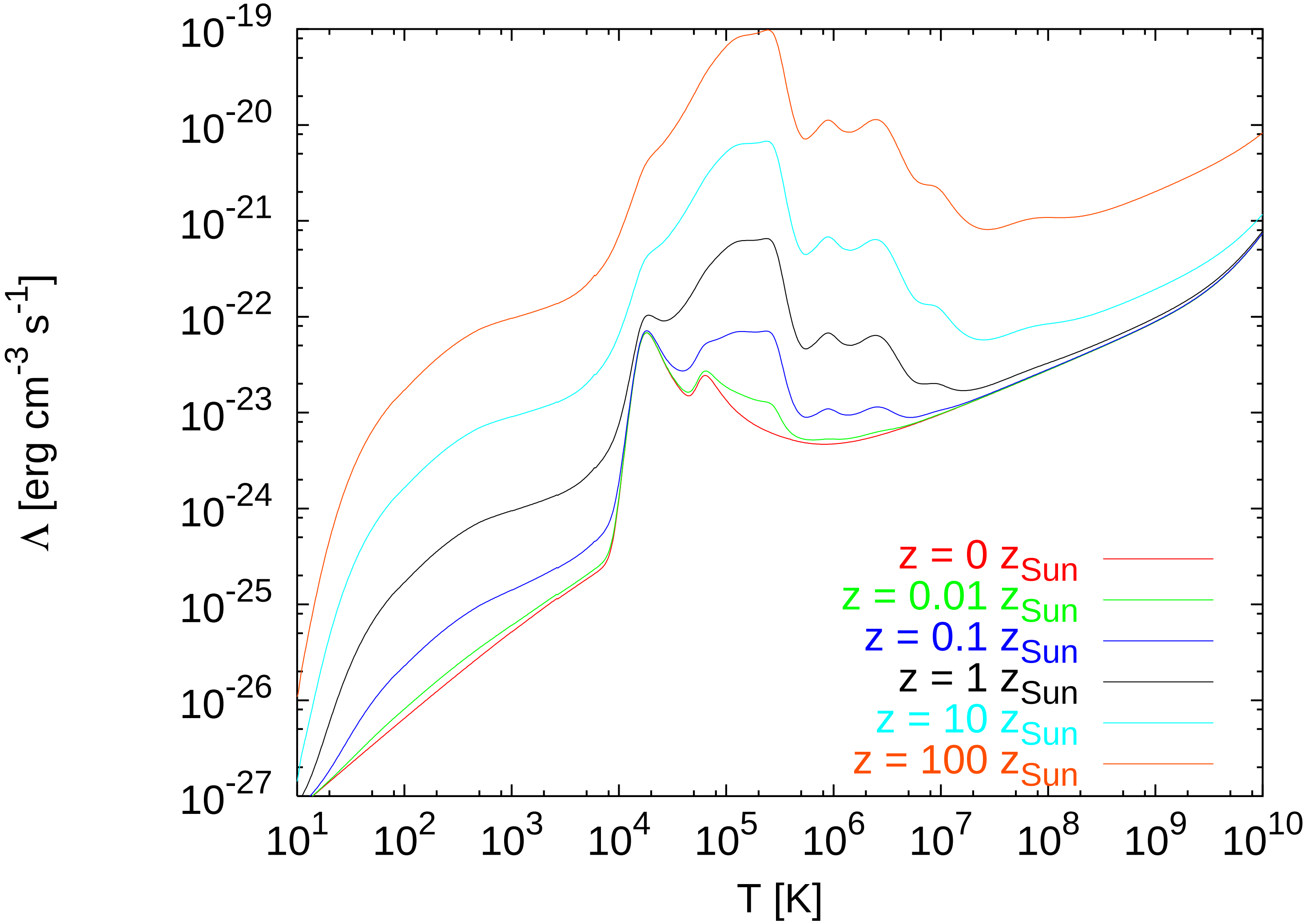




The cooling function

The energy lost by a radiation: $\frac{dE}{dt} = -\rho^2 \Lambda(T)$,

where ρ ... density, T ... temperature

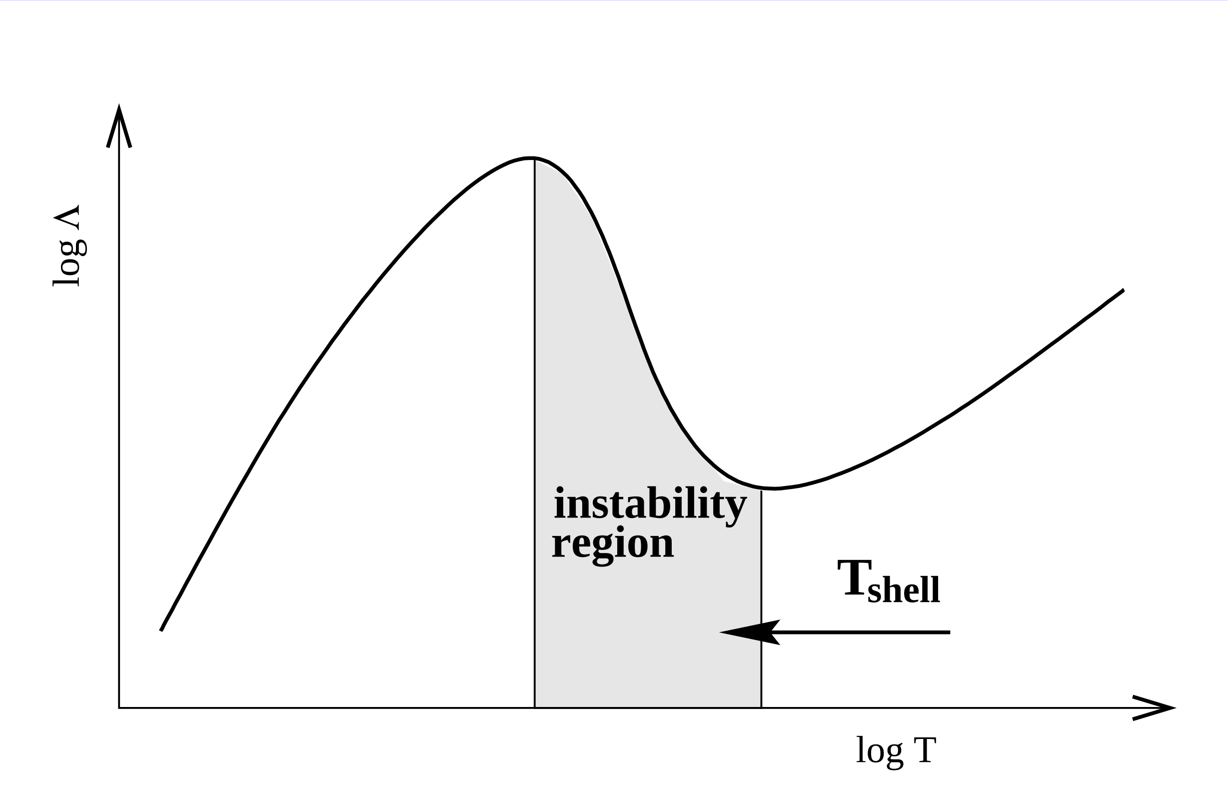


Thermal instability: basic idea

Instability region in the cooling function: Λ falls steeply with the increasing temperature.

Positive feedback: decrease of T $\rightarrow$ more cooling $\rightarrow$ further decrease of T

When T_{shell} enters the instability region, the shell collapses very quickly



Collapse of the shell

- **Catastrophic cooling in SNR (Falle, 1981)**
- **simulation with abrupt energy insertion (SN)**
- **initial phase: almost adiabatic $\rightarrow$ blastwave problem $\rightarrow$ analytical solution known (Sedov, 1959),** $R(t) \sim \left(\frac{Et^2}{\rho_0}\right)^{(1/5)}$

Parameters of the simulation:

energy: 10^{51} erg **(1 SN)**

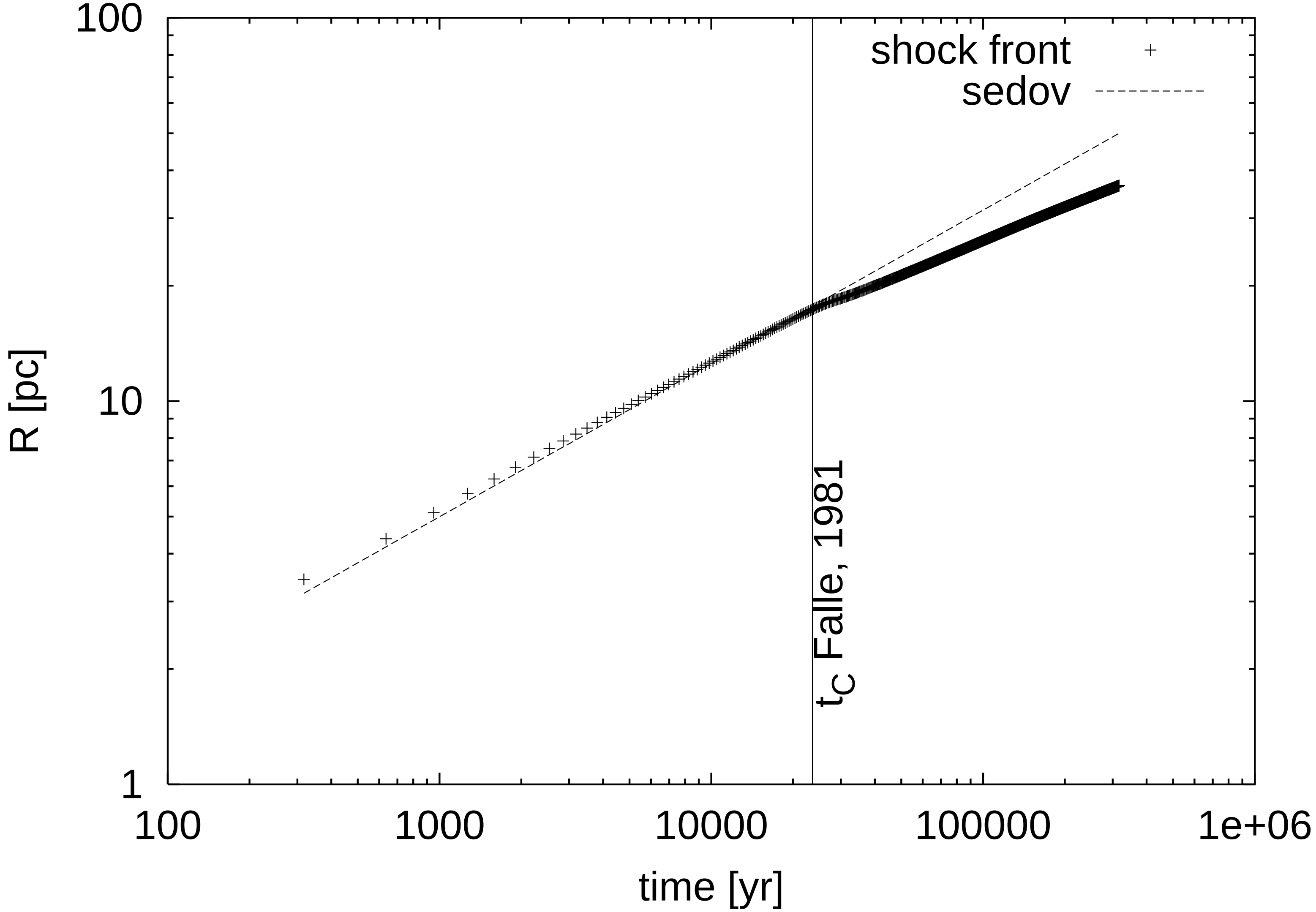
ambient medium density: $1.3\ \mathrm{m_H cm^{-3}}$

temperature: 10^4 K

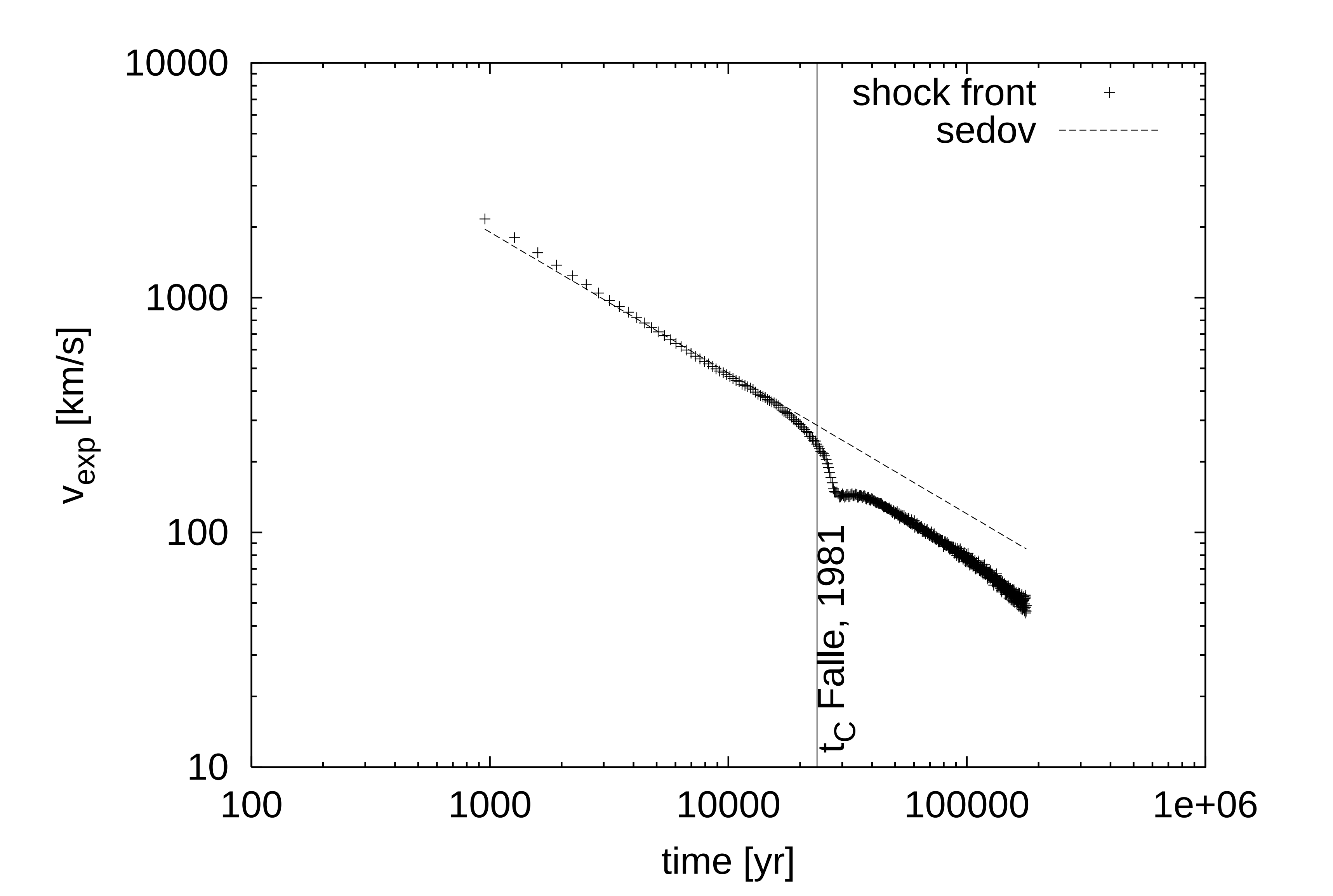
resolution: $400 \times 1 \times 1$

Collapse time: $t_C = 2.17 \cdot 10^4$ yr

Catastrophic cooling: position of the shock front



Catastrophic cooling: velocity of the shock



Poisson solver

Poisson solver: $\rho(x_1, x_2, x_3) \rightarrow \Phi(x_1, x_2, x_3)$

Poisson equation $\nabla^2\Phi = 4\pi G\rho$ **must be fulfilled**

Discretization of the PE in general coordinates:

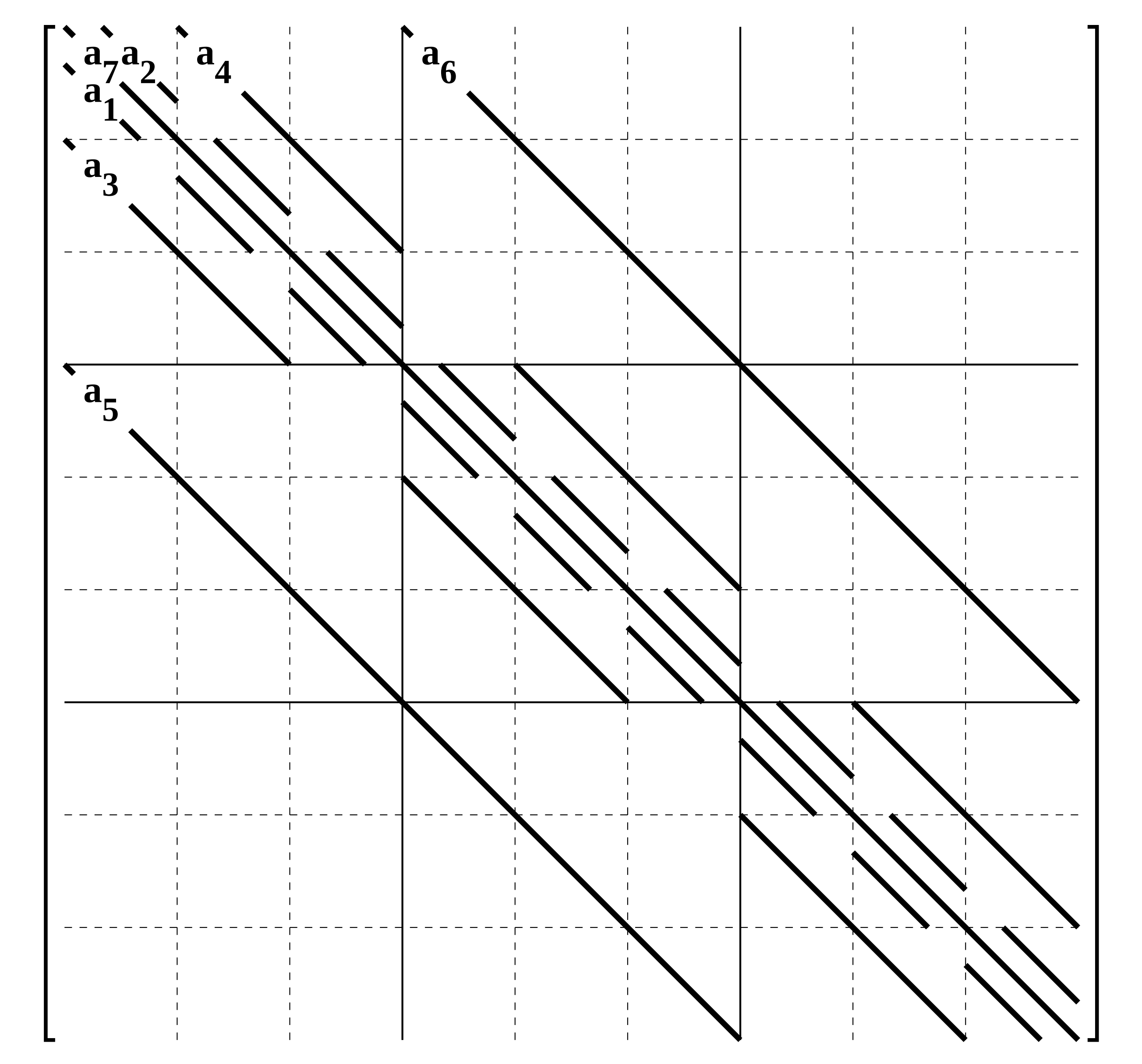
$$\begin{aligned}\nabla^2\Phi &= a_1\Phi_{i-1,j,k} + a_2\Phi_{i+1,j,k} + a_3\Phi_{i,j-1,k} \\ &+ a_4\Phi_{i,j+1,k} + a_5\Phi_{i,j,k-1} + a_6\Phi_{i+1,j,k+1} \\ &+ a_7\Phi_{i,j,k} = 4\pi G\rho_{i,j,k}\end{aligned}$$

this algebraic equation must be fulfilled in each cell

$\Rightarrow$ **we have the set of** $L \times M \times N$ **equations for a simulation on the** $L \times M \times N$ **grid**

$$Ax = b$$

Poisson solver (Poisson equation matrix)



Poisson solver (DADI)

DADI = Dynamic Alternating Direction Implicit

- **splits matrix A in two parts:** $A = E - F$
  E - tridiagonal matrix, easy to invert
  F - remaining part

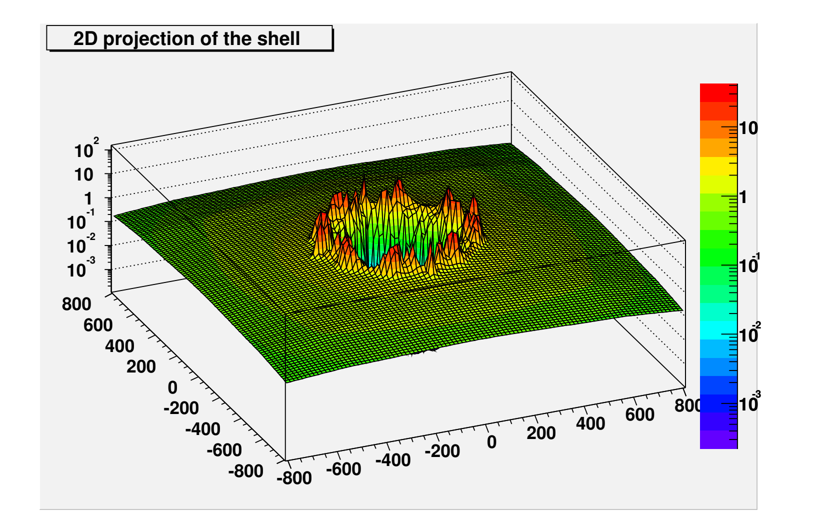
  $(E - F)x = b \quad \rightarrow \quad x^{n+1} = E^{-1}(Fx^n + b)$
- **alternating direction** (the way, how the equations are sorted) **ensures the convergence. (ADI method)**

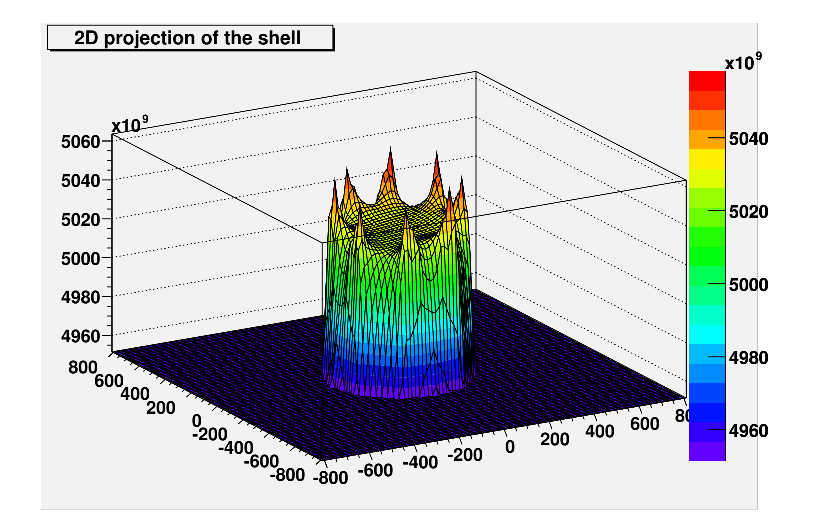
<u>**DADI method**</u>

- **accelerates the convergence**
- **equilibrium solution of the heat transfer eq.:**
  $\frac{\partial \Phi}{\partial g} = \nabla \Phi - 4\pi G\rho$, **where** g **is a virtual time**
- **one virt. timestep is made by the ADI method**

Example simulation with the gravity

- simulation in the cartesian coordinates (100x100x100 grid) with the DADI Poisson solver
- fragments have artificial origin (numerical noise produced by Cartesian coord. in cooperation with the thermal instability)





up: distribution of the density – 2D cut through the computational area

left: the detail of the gravitational potential in the shell

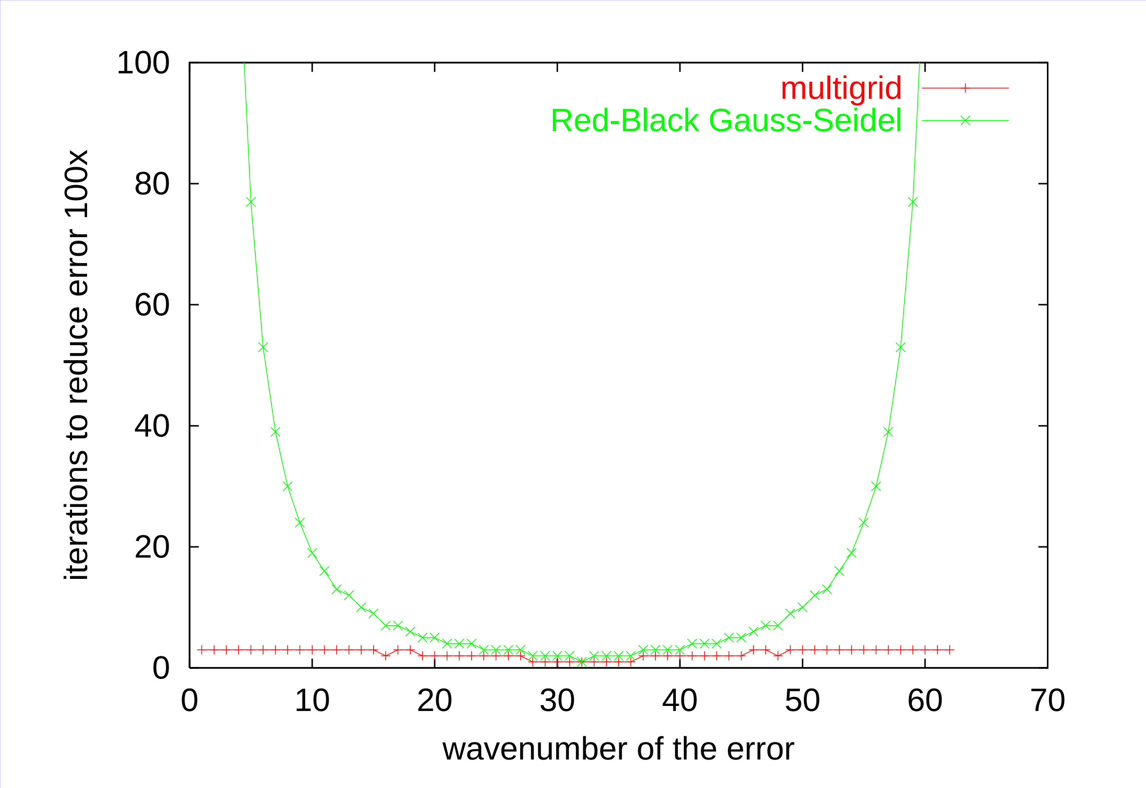
Poisson solver: Multigrid method

- problem with the DADI (and all standard iterative methods): they are absolutelly ineffective in reducing the errors with long wavelengths
- solution – multigrid methods
- basic idea (very briefly) – V-cycle:

1. relax on the fine grid
2. restrict the problem to the coarser grid and do a relaxation
3. ...
4. solve the set of equations on the coarsest grid (exactly)
5. interpolate to a finer grid and do arelaxation
6. ...
7. final relaxation of the finest grid

Multigrid vs. standard method

relaxation on 64x64x64 grid – multigrid vs. Red-Black Gauss-Seidel method
number of iterations to reduce error 100x
(RGBS needs more than 1000 iterations to reduce error with $k = 1$)



Next steps

- **computations in the spherical coordinates with the multi-grid solver**
- **mass spectrum of fragments**
- **decomposition into spherical harmonics**

References

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