PhD thesis:

Gravitational instabilities of expanding shells

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Plans:

(Magneto)hydrodynamic simulations of protoplanetary disks

in collaboration with Prof. Michał Różyczka

Warsaw node of The Network

HI shells

- **basic characteristics:**
  $R \sim 10$ pc $-$ 2 kpc, $E \sim 10^{51} - 10^{54}$ erg, $v_{exp} \sim 5 - 30$ km/s
- **Observations:**
  MW: Heiles (1979); recent obs.: McClure-Griffiths et al. (2002)
  external galaxies: LMC (Kim, 1998), SMC (Stanimirovic, 1999)
  M31, M33, M101, HoII, IC10
- **origin - SN explosion, OB association, group of OB as., encounter with HVC or a dwarf galaxy, GRB**
- **Why are HI shells interesting?**
  - **Propagating Star Formation:**
    OB as. → exp. HI shell → fragments → new stars
  - **Disk Halo Connection**

Example of the HI shell

The galactic HI shell GS59.9-1.0+38 observed by Ehlerová et al (1997) with 100 m Effelsberg radio-telescope. Left: The *lb*-cut of the data-cube in velocity channel $v = 36.6$ km/s. Right: The spectrum through the center of the shell.

Gravitational instability - the linear analysis

Elmegreen (1994)

Assume a cold and thin shell of the radius r expanding with the velocity v into the medium of the density ρ_0. HD equations and the Poisson equation on the shell surface:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \Omega) = v \rho_0,$$

$$\frac{\partial}{\partial t}(\Sigma\Omega) + (\Omega \cdot \nabla)\,\Sigma\Omega = -\Sigma\nabla\Phi - c^2 \nabla\Sigma$$

$$\Delta\Phi = 4\pi G \Sigma \delta(R - r)$$

- Σ – surface density of the shell
- Ω – 2D angular velocity of the surface flows
- c – sound speed in the shell
- Φ – gravitational potential

→ **linearization, stability analysis of the Fourier modes**
(parametrized by the dimensionless wavenumber $\eta = kr$):

$$\omega(\eta) = -\frac{3v}{r} + \sqrt{\frac{v^2}{r^2} + \frac{2\pi G \Sigma_0 \eta}{r} - \frac{c^2 \eta^2}{r^2}}$$

Gravitational instability - the quadratic terms

Wünsch & Palouš (2001)

- similar approach as Fuchs (1996) – uniformly rotating disk
- quadratic terms couple Fourier modes inclined at an angle 60 deg
- coupling weaker than in the case of the disk → should be tested numerically

up: the surface density which appropriates to the triplet of interacting modes

left: the velocity field of the surface flows of mass in the same case

Mass spectrum of fragments

- can be derived from the dispersion rel. $\omega(\eta)$ (from the linear analysis)
- the fragmentation integral:

$$I_f(\eta, t) = \int_{t_B}^{t} \omega(\eta, t')dt' \ ,$$

where t_B is the time when the instability begins

- the mass spectrum:

$$dN \sim I_f(\eta, t) \times \frac{r^2}{\lambda^2} d\eta$$

$$\rightarrow dN \sim m^{-\alpha} dm \ ,$$

where α is mass spectrum index

expanding shell yields the mass spectrum index $\alpha = 2.35$ at high mass end (very close to the stellar IMF) independently on the parameters of the shell and on the stage of its evolution, conf. to other physical situations

Mass spectrum dependency

The mass spectrum of fragments depends on L**,** c **and** n_0**. The most important parameter determining the mass range of fragments is** n_0**.** $n_0 \sim 0.1 - 1 cm^{-3} \rightarrow m_{\rm frag} \sim 10^4 - 10^7 M_\odot$**;** $n_0 \sim 10^6 - 10^7 cm^{-3} \rightarrow m_{\rm frag} \sim 10^{-1} - 10^3 M_\odot$**.**

Numerical model

- **based on ZEUS3D hydrocode (Stone & Norman, 1992)**
- **radiative cooling:** $\frac{de}{dt}\Big|_{r.c.} = -\rho^2 \Lambda(T, Z)$
- **artificial (turbulent or magnetic) pressure (prevents shell from collapsing out of resolution)**
- **self-gravity: multigrid Poisson solver Šemík**
- **the energy is inserted into the central area at rate** L**, ambient medium has density** $\rho_0 = n_0 \mu$ **and temperature** T_0

Poisson solver: Šemík

- **Poisson equation** $\nabla^2\Phi = 4\pi G\rho$
  → huge set of linear equations $A\mathbf{\Phi} = \mathrm{const}\cdot\rho$
- **can be solved iteratively (advantage: the solution from the previous HD time-step can be used)**
- **all standard iterative methods are inefficient in eliminating long modes of error**
- **multigrid methods: eliminate the long modes of error on coarser grids**
- **very efficient:** $O(N)$**, 2-3 iteration steps per 1 HD step**

Numerical vs. analytical solution

Test of the linear analysis: initial perturbation of the surface density is a single mode. We follow the growth of perturbations and compare it to the growth rate given by the linear analysis.

Perturbations grow a bit faster in the numerical solution ($t_{\mathrm{frag,num}} \sim 0.9 t_{\mathrm{frag,lin}}$): cores of fragments are colder, while the linear analysis assumes isothermal shell.

Mass spectrum of fragments

Initial conditions: random perturbations of the surface density at the beginning of the grav. instability. Fragments are detected and the mass spectrum is made. The mass spectrum index is $\alpha = 2.4$.

The uncertainty of α is still quite high due to insufficient resolution, for various resolutions and ways of detection of fragments $\alpha \sim 2.2 - 3.0$.

(M)HD simulations of protoplanetary disks

- **we plan to test the consistence of the layred disk model**
- **layered disks (weakly ionized):**
  surface (active) layers are well coupled to the mag. field, while the midplane is decoupled $\rightarrow$ dead zone
- **the viscosity leads to the heating of the active disk layers**
- **the boundary of the dead zone may be heated up from the active layers and the active part of the disk may spread**