RHD simulations of layered disks

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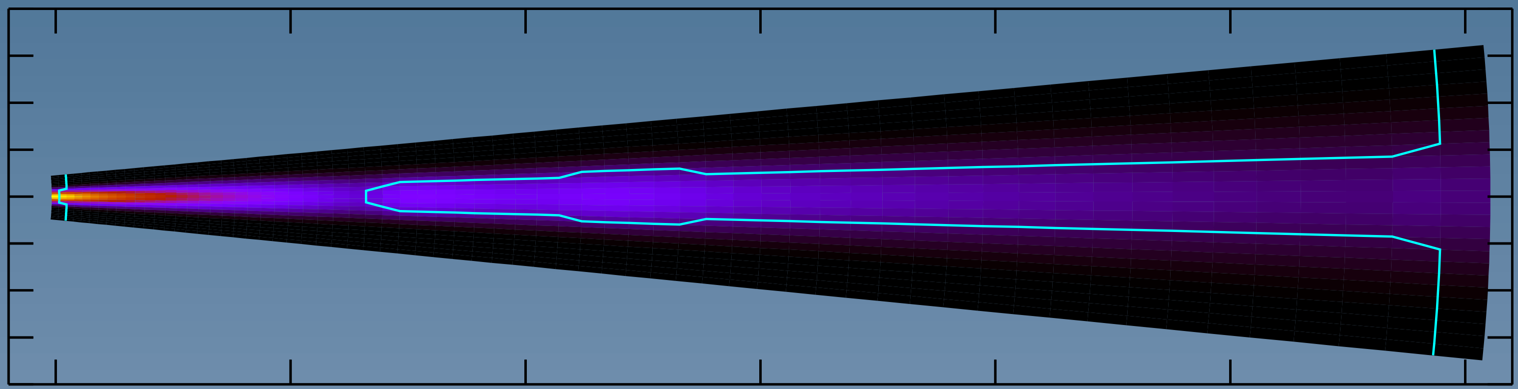


Outline:

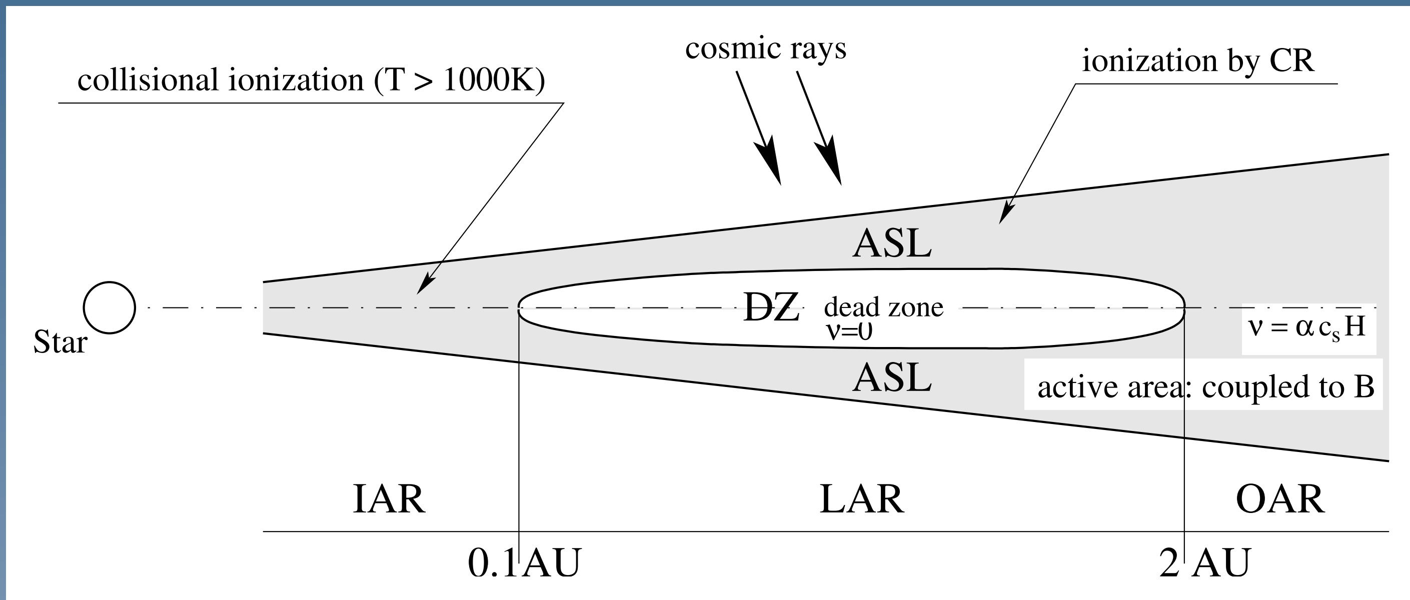
1 Analytical description of LD

2 Numerical model

3 Ring instability

4 Evolution of the dead zone

Layered-disk: basic idea (Gammie, 1996)



- angular momentum transfer - MRI (Balbus & Hawley, 1991)
- parts of the disk are not ionized enough to be well coupled to the magnetic field
- inner active region (IAR) - collisional ionization
- layered accretion region (LAR) - surface active layers (ASL) ionized by cosmic rays shield the dead zone (DZ) near the mid-plane
- outer active region (OAR) - low surface density, CR are able to ionize whole disk

Layered-disk: physical processes

- MRI occurs for: $Re_M \equiv \frac{V_A H}{\eta} > 1$
- Alfven velocity related to α-viscosity: $V_A = \alpha^{1/2} c_s$
- resistivity η related to the ionization degree $x = n_e/n_H$:

$$\eta = 6.5 \times 10^3 x^{-1} \mathrm{cm}^2 \mathrm{s}^{-1}$$

- using $H = c_s/\Omega$ magnetic Reynolds number:

$$Re_M = 7.4 \times 10^{13} x \alpha^{1/2} \left(\frac{R}{AU}\right)^{3/2} \left(\frac{T}{500K}\right) \left(\frac{M}{M_\odot}\right)^{-1/2}$$

- collisional ionization: $x = x(\rho, T)$ (Umebayashi, 1983)

$$x \sim \log(\rho), \qquad x(T) = \begin{cases} 10^{-16} & \text{for} \quad T \leq 800 \ K \\ 10^{-13} & \text{for} \quad T \sim 900 \ K \\ 10^{-11} & \text{for} \quad T \geq 1000 \ K \end{cases}$$

- CR ionization: stopping depth $\Sigma_0 \sim 100 \ \mathrm{g/cm}^2$

(Umebayashi & Nakano, 1981)

$$x = \left(\frac{\zeta}{\beta n_H}\right)^{1/2} = 1.6 \times 10^{-12} \left(\frac{T}{500\mathrm{K}}\right)^{1/4} \left(\frac{\zeta}{10^{-17}\mathrm{s}^{-1}}\right)^{1/2} \left(\frac{n_H}{10^{13}\mathrm{cm}^{-3}}\right)^{-1/2}$$

Layered disk: basic equations for ASL

- mass and angular momentum conservation:

$$\dot{M} = 6\pi r^{1/2} \frac{\partial}{\partial r}(2\Sigma_a \nu r^{1/2})$$

- α-viscosity: (Shakura & Sunayev, 1973)

$$\nu = \alpha c_s H_a$$

- energy released by accretion dissipated locally:

$$\frac{9}{4}\Sigma_a \nu \Omega^2 = \sigma T_e^4$$

- emission is optically thick: (Hubeny, 1990)

$$T_c^4 = \frac{3}{8}\Sigma_a \kappa(\rho, T_c) T_e^4$$

- vertical hydrostatic equilibrium:

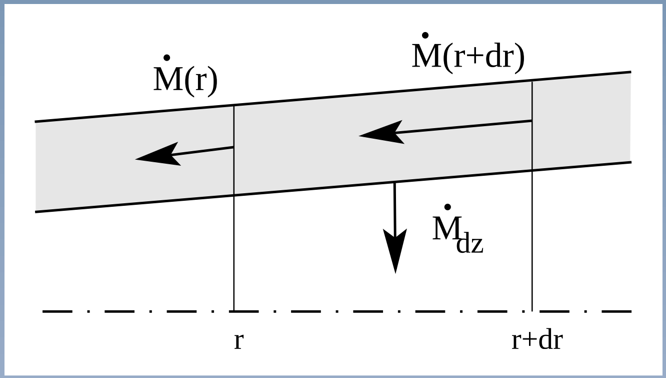
$$c_{\rm s}^2 = H_{\rm a}(H_{\rm a} + H_{\rm DZ})\Omega^2$$

- disk is geometrically thin:

$$\Omega = \left(\frac{GM}{R}\right)^{1/2}$$

Layered disk: analytical solution

- Standard solution (Gammie, 1996): $H_{\rm DZ} \rightarrow 0$
- radial profiles $\dot{M}$, T_e, T_c, H_a, ... $\rightarrow$ power-laws
- exponents dependent on the opacity $\kappa = \kappa(\rho, T)$
- α and Σ_a are the free parameters
- $\Sigma_a = \rm const \quad \Rightarrow \quad \dot{M} = \dot{M}(r)$ increasing with r
- accumulation of mass in DZ:
  $\dot{\Sigma}_{\rm DZ} = \frac{1}{2\pi r}\frac{\partial \dot{M}}{\partial r}$
- accretion cannot be steady - when $\Sigma_{\rm DZ}$ is high enough, DZ mass is accreted in an outburst like event
  $\Rightarrow$ suggested as mechanism for FU Orionis outbursts (Gammie, 1996)



Numerical model

- RHD code TRAMP: (Klahr et al., 1999)

  (Three-dimensional RAdiation-hydrodynamical Modeling Project)

- solves set of Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\rho \mathbf{v} = -\nabla P - \rho \nabla \Phi + \nabla \cdot \mathbf{T} \qquad P = \frac{kT}{\mu m_H}\rho$$

$$c_v \rho \left[\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T\right] = -P \nabla \cdot \mathbf{v} + \mathbf{T} : (\nabla \mathbf{v})$$

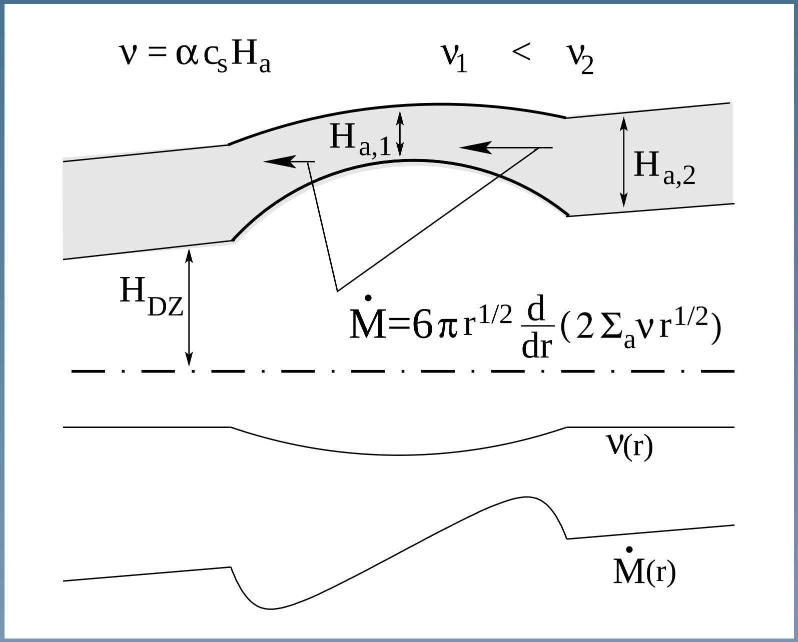
- at the end of time-step: radiation transfer

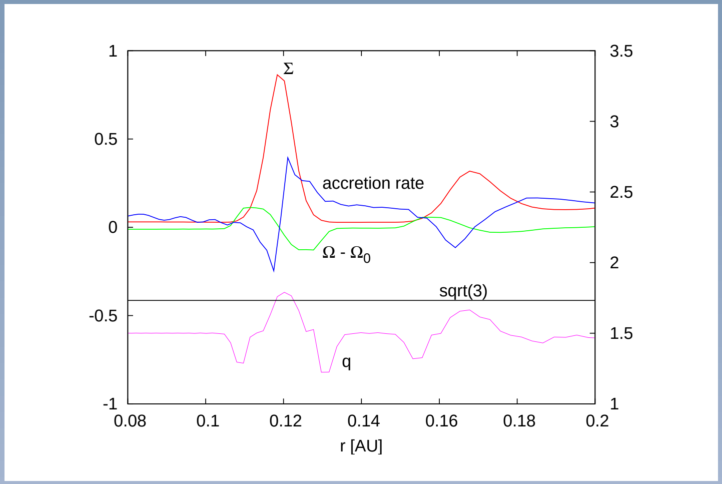
$$\frac{\partial E_r}{\partial t} = -\nabla \cdot \mathbf{F} \;, \quad \text{where} \quad E_r = aT^4 \;, \quad \mathbf{F} = -\frac{\lambda c}{\rho \kappa} \nabla E_r$$

- 2D axially symmetric in spherical (r, θ) coords.
- initial conditions: analytical model (vertically isothermal)
- viscosity:
  - ▷ $\alpha = 0.01$ - *surface layers* ($\Sigma_a = 100\,\mathrm{g\,cm^{-2}}$) *and inner region* ($T > 1000\mathrm{K}$)
  - ▷ $\alpha = 0$ - *elsewhere (dead zone)*

Ring instability

- dead zone decomposes into rings
- ring instability mechanism:
  - ▷ *thickness of surface layer $H_{\rm a}$ depends on the dead zone thickness $H_{\rm DZ}$ (due to different vertical gravity)*
  - ▷ *$H_{\rm a}$ is smaller in the ring-like perturbation $\Rightarrow$ ν is smaller there, too*
  - ▷ *$\dot{M}$ depends on derivative of ν $\Rightarrow$ it is smaller in inner edge and higher in outer edge of the ring*
  - ▷ *enhanced mass accumulation in the ring $\Rightarrow$ positive feedback*





- rings may work as traps for the dust $\rightarrow$ formation of planets
- rings may decay due to the hydrodynamic instability, if $q > \sqrt{3}$ $(\Omega \sim r^{-q})$

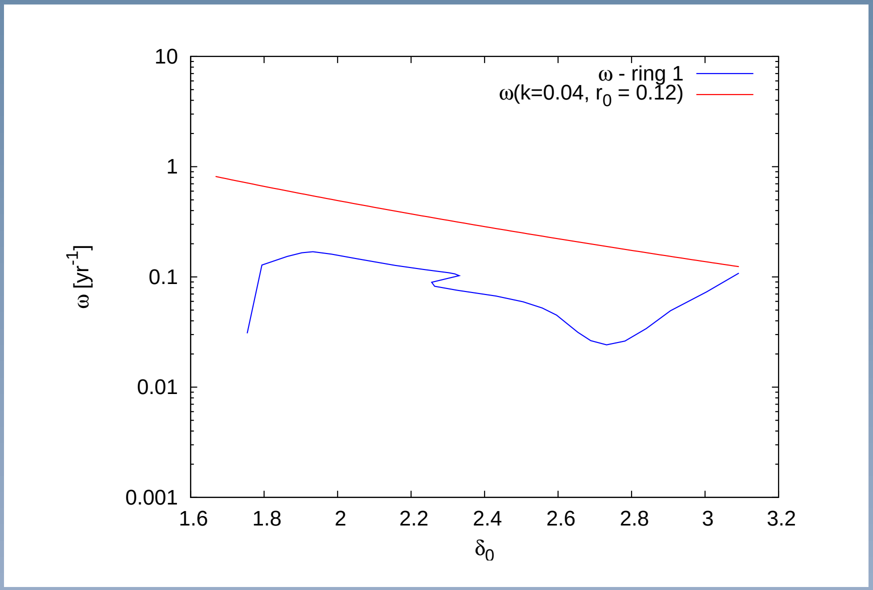
(Papaloizou & Pringle, 1985)

Ring instability - analytical description

- thickness of the dead zone important for $\dot{M}$ (Huré, 2002)
- dimensionless disk thickness: $\delta = \frac{H_a + H_{\rm DZ}}{H_a}$
- rotational velocity corrected to mid-plane pressure:
$$\Omega^2 = \Omega_0^2 + \frac{1}{\rho_m}\frac{\partial P_m}{\partial r}$$
- ring-like perturbation of δ:
$$\delta = \delta_0 + \delta_k \cos(kR), \text{ where } R = r - r_0$$
δ_0 - unperturbed disk thickness, r_0 - position of the ring,
k - wavenumber
- inserting into equations of layered disk, linearization
- $\rightarrow$ perturbation growth rate given by the equation:
$$\dot{\delta_k} = \omega(r_0, \delta_0, k)\delta_k$$

Ring instability - dispersion relation

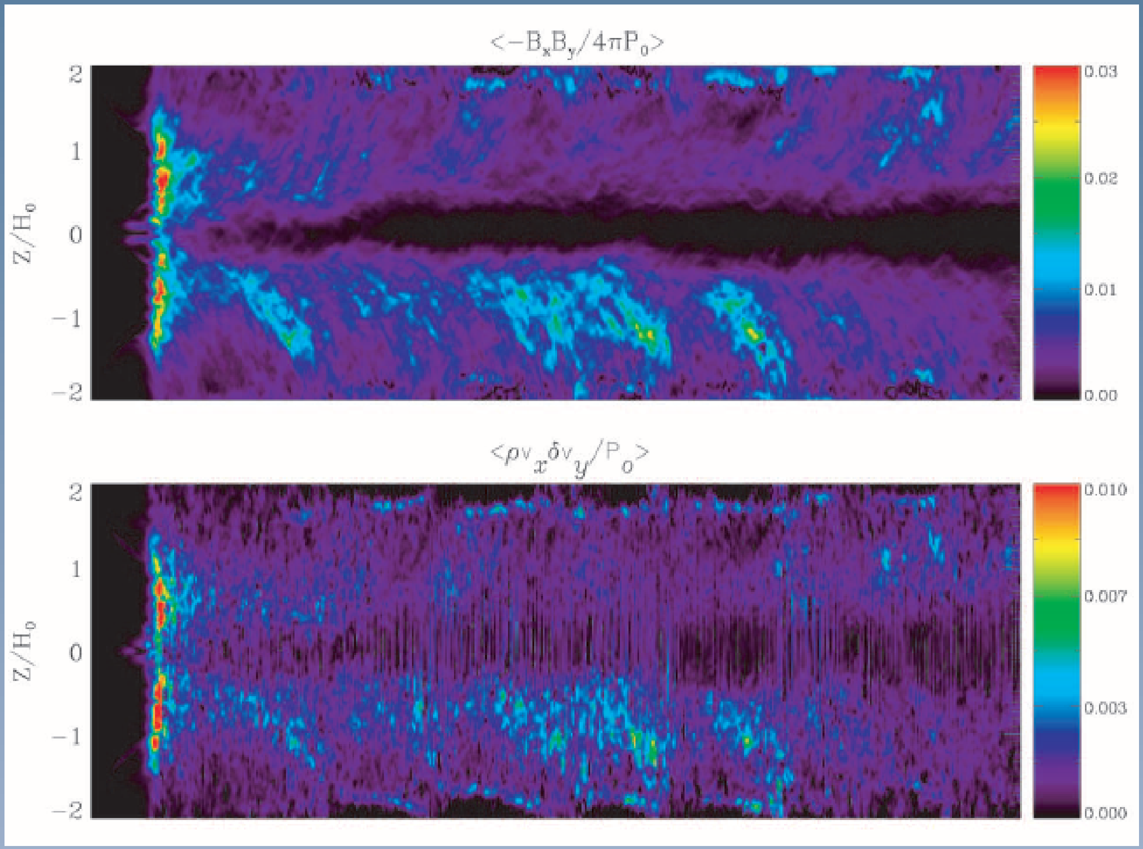
- ω diverges for $k \rightarrow \infty$ ($\lambda \rightarrow 0$) $\Rightarrow$ the thinnest rings are the most unstable
- radial extent of rings is given by thermal motion of particles $\lambda_0 \sim \frac{c_s}{\Omega}$
- radial extent of rings in simulation in agreement with analytical model ($\sim$ vertical thickness)
- growth rate is smaller in simulation because the radial transfer of heat makes the temperature profile shallower



Layered-disks: MRI simulations

(Fleming & Stone, 2003)

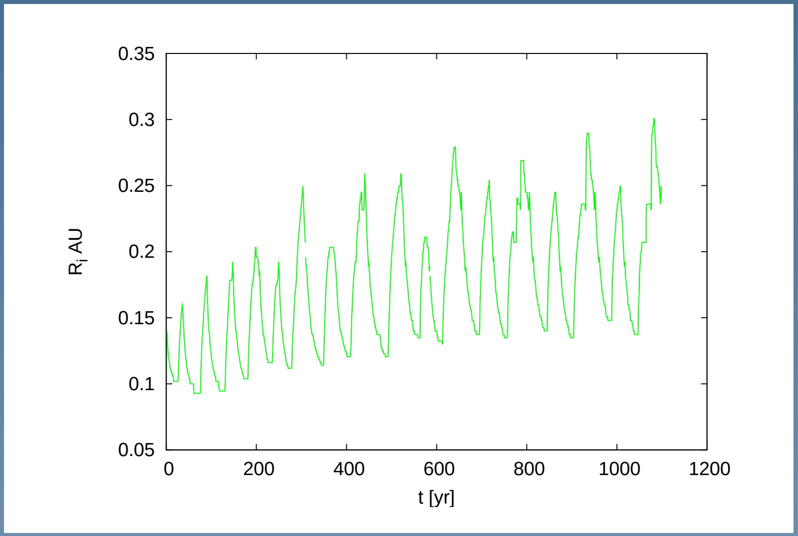
- indications for a small viscosity in the dead zone
- non-ideal MHD ($\eta \neq 0$), shearing box, isothermal EOS

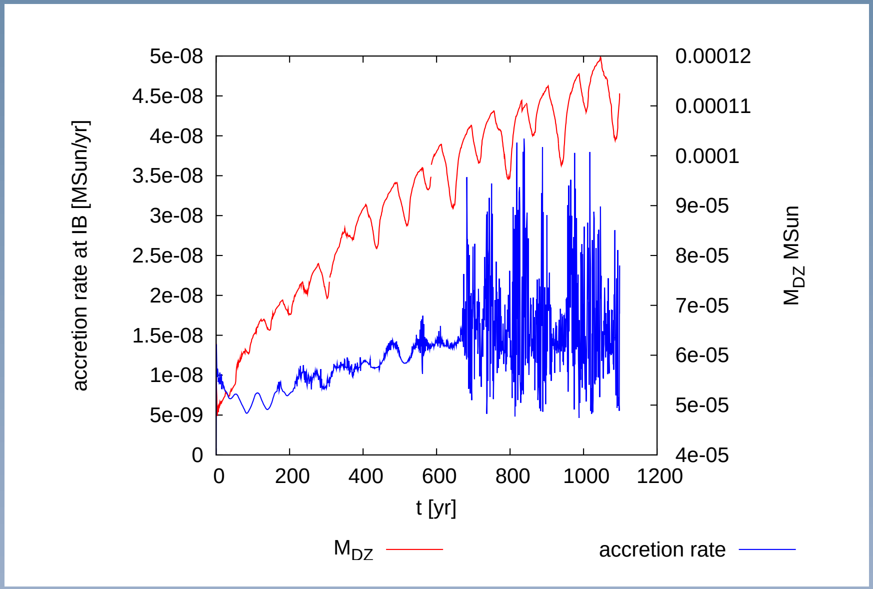


- Maxwell stress vanishes in DZ - MHD turbulence decays
- Reynolds stress non-zero in DZ - HD turbulence survives due to perturbations from active layers (10% of Maxwell s.)

Evolution of the dead zone

- inner edge of DZ oscillates
- timescale $\sim 10 - 100\mathrm{yr}$
- DZ continuously depleted from inner edge





- small changes in $\dot{M}$ (amplitude $0.5 - 1 \times 10^{-8}$ $M_\odot yr^{-1}$)
- mass stored in the dead zone grows

Conclusions

- the dead zone with zero viscosity is a subject of the ring instability
- the instability occurs since the local enhancement of the disk thickness leads to a drop of the viscosity and the accretion rate profile which increases the original disk thickness enhancement

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- the dead zone with zero viscosity is a subject of the ring instability
- the instability occurs since the local enhancement of the disk thickness leads to a drop of the viscosity and the accretion rate profile which increases the original disk thickness enhancement
- in the case of small viscosity in DZ the oscillations of the inner edge occur
- mass is continuously removed from the inner part of the dead zone - not consistent with the sudden huge increase of accretion rate due to the ignition of mass accumulated in the dead zone (FU Ori outburst)

References

Balbus & Hawley, 1991, ApJ, 376, 214
Fleming & Stone, 2003, ApJ, 585, 908
Gammie, 1996, ApJ, 457, 355
Huré, J.-M., 2002, PASJ, 54, 775
Hubeny, 1990, ApJ, 351, 632
Klahr, H., Henning, T., Kley, W., 1999, ApJ, 514, 325
Papaloizou, J. C. B., Pringle, J. E., 1985, MNRAS, 213, 799
Shakura & Sunayev, 1973, A&A, 24, 337
Umebayashi, 1983, Prog. Theor. Phys., 69, 480
Umebayashi & Nakano, 1981, PASJ, 33, 617