2D models of layered protoplanetary discs

R. Wünsch, H. Klahr and M. Różyczka, 2005, MNRAS, 362, 361
The ring instability

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The effect of a residual viscosity in the dead zone

Outline:

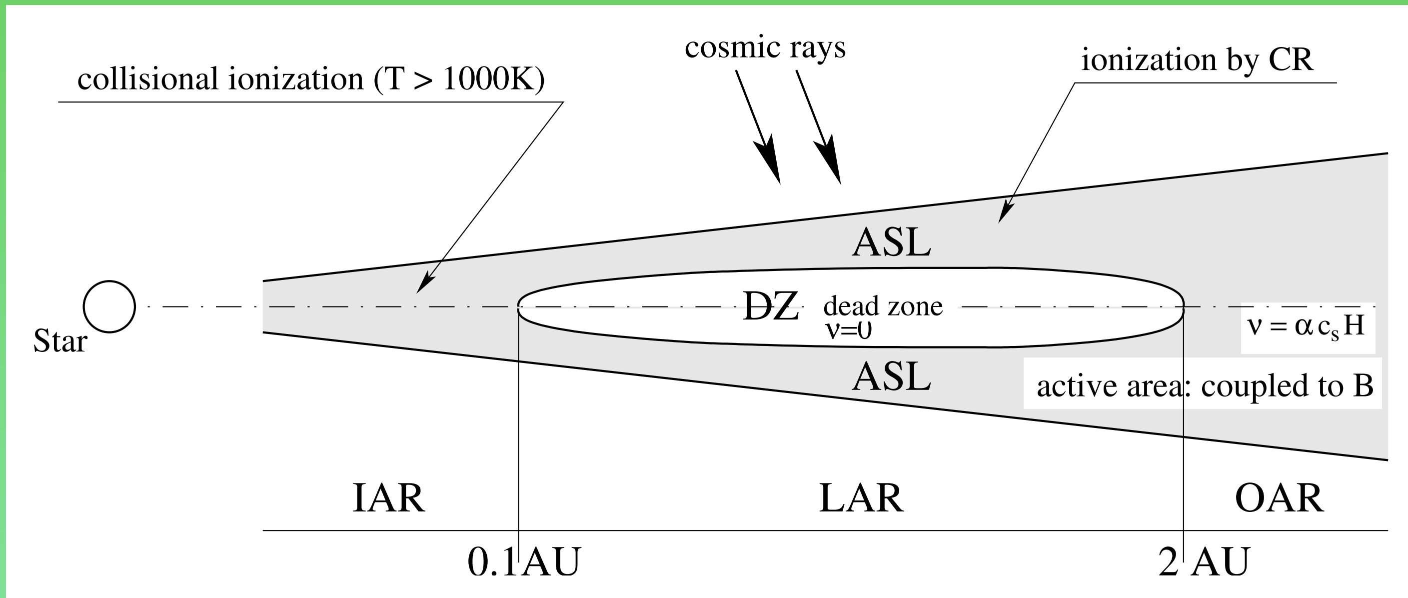
1 Introduction of the layered disc model

2 Numerical code

3 Ring instability

4 Residual viscosity in the dead zone

Layered-disc: basic idea (Gammie, 1996)



- angular momentum transfer - MRI (Balbus & Hawley, 1991)
- parts of the disc are not ionized enough to be well coupled to the magnetic field
- inner active region (IAR) - collisional ionization
- layered accretion region (LAR) - surface active layers (ASL) ionized by cosmic rays shield the dead zone (DZ) near the mid-plane
- outer active region (OAR) - low surface density, CR are able to ionize whole disc

Layered-disc: physical processes

- MRI occurs for: $Re_M \equiv \frac{V_A H}{\eta} > 1$
- Alfven velocity related to α-viscosity: $V_A = \alpha^{1/2} c_s$
- resistivity η related to the ionization degree $x = n_e/n_H$:

$$\eta = 6.5 \times 10^3 x^{-1} \mathrm{cm}^2 \mathrm{s}^{-1}$$

- using $H = c_s/\Omega$ magnetic Reynolds number:

$$Re_M = 7.4 \times 10^{13} x \alpha^{1/2} \left(\frac{R}{AU}\right)^{3/2} \left(\frac{T}{500K}\right) \left(\frac{M}{M_\odot}\right)^{-1/2}$$

- collisional ionization: $x = x(\rho, T)$ (Umebayashi, 1983)

$$x \sim \log(\rho), \qquad x(T) = \begin{cases} 10^{-16} & \text{for} \quad T \leq 800\ K \\ 10^{-13} & \text{for} \quad T \sim 900\ K \\ 10^{-11} & \text{for} \quad T \geq 1000\ K \end{cases}$$

- CR ionization: stopping depth $\Sigma_0 \sim 100\ \mathrm{g/cm}^2$

(Umebayashi & Nakano, 1981)

$$x = \left(\frac{\zeta}{\beta n_H}\right)^{1/2} = 1.6 \times 10^{-12} \left(\frac{T}{500\mathrm{K}}\right)^{1/4} \left(\frac{\zeta}{10^{-17}\mathrm{s}^{-1}}\right)^{1/2} \left(\frac{n_H}{10^{13}\mathrm{cm}^{-3}}\right)^{-1/2}$$

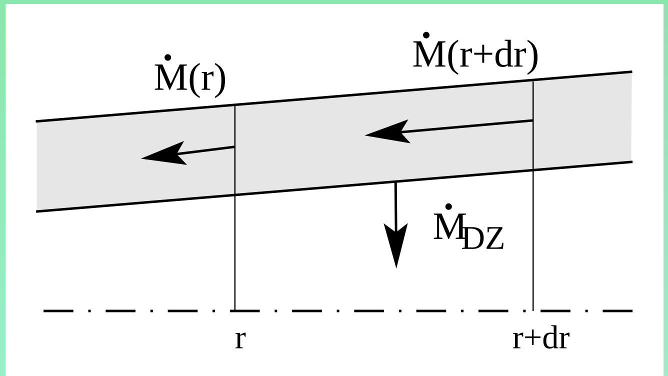
Properties of the layered accretion

- description of the layered accretion region:

$$\dot{M} = 6\pi r^{1/2}\frac{\partial}{\partial r}(2\Sigma_a \nu r^{1/2}), \qquad \nu = \alpha c_{s,i} H_a, \qquad \frac{9}{4}\Sigma_a \nu \Omega^2 = \sigma T_e^4$$

$$T_i^4 = \frac{3}{8}\Sigma_a \kappa(\rho_i, T_i) T_e^4, \qquad c_{s,i}^2 = H_\mathrm{a}(H_\mathrm{a} + H_\mathrm{DZ})\Omega^2$$

- solution: $\dot{M}(r)$, $T_e(r)$, $T_i(r)$, $\ldots \rightarrow$ power-laws, exponents dependent on the opacity $\kappa = \kappa(\rho_i, T_i)$
- $\Sigma_a = \mathrm{const} \;\; \Rightarrow \;\; \dot{M} = \dot{M}(r)$ increasing with r
- accumulation of mass in DZ: $\dot{\Sigma}_\mathrm{DZ} = \frac{1}{2\pi r}\frac{\partial \dot{M}}{\partial r}$
- accretion cannot be steady - when Σ_DZ is high enough, DZ mass is accreted in an outburst like event $\Rightarrow$ suggested as mechanism for FU Orionis outbursts (Gammie, 1996)



Numerical model

- RHD code TRAMP: `(Klahr et al., 1999)`

  (Three-dimensional RAdiation-hydrodynamical Modeling Project)

- solves set of Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \rho \mathbf{v} = -\nabla P - \rho \nabla \Phi + \nabla \cdot \mathbf{T} \qquad P = \frac{kT}{\mu m_H} \rho$$

$$c_v \rho \left[\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T \right] = -P \nabla \cdot \mathbf{v} + \mathbf{T} : (\nabla \mathbf{v})$$

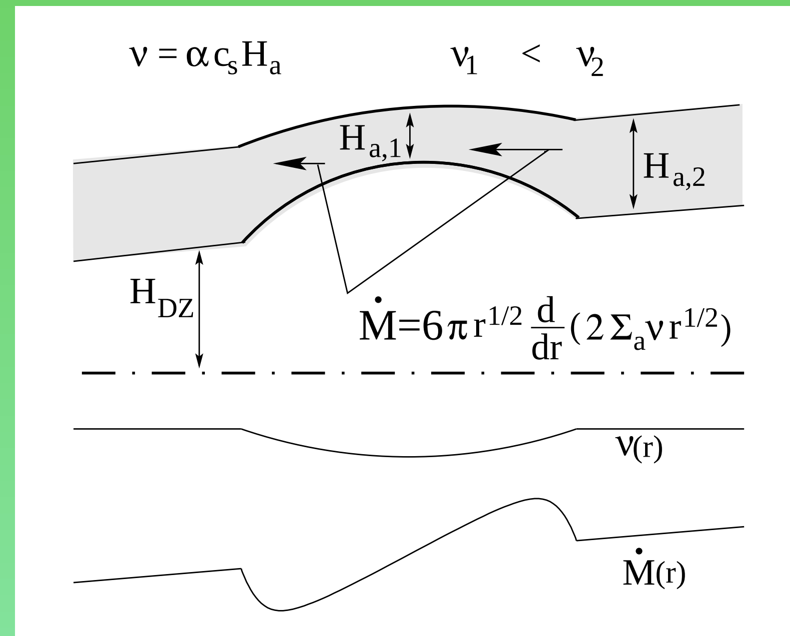
- radiation transfer: flux limited diffusion approximation

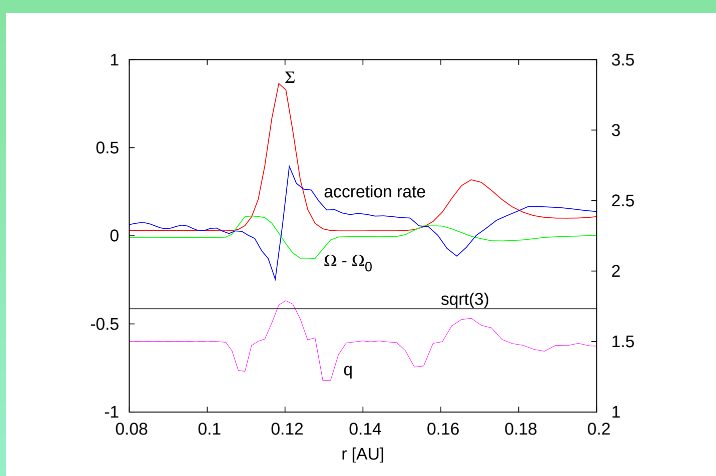
$$\frac{\partial E_r}{\partial t} = -\nabla \cdot \mathbf{F} \ , \quad \text{where} \quad E_r = aT^4 \ , \quad \mathbf{F} = -\frac{\lambda c}{\rho \kappa} \nabla E_r$$

- 2D axially symmetric in spherical (r, θ) coords.
- initial conditions: analytical model (vertically isothermal)
- viscosity:
  - ▷ $\alpha_a = 10^{-2}$ - *surface layers* ($\Sigma_a = 100 \, \mathrm{g\,cm^{-2}}$) *and inner region* ($T > 1000 \mathrm{K}$)
  - ▷ $\alpha_{DZ} = 0, 10^{-4}, 10^{-3}$ - *elsewhere (dead zone)*

Ring instability - mechanism

- dead zone decomposes into rings
- ring instability mechanism:
  - ▷ *thickness of surface layer H_a depends on the dead zone thickness H_{DZ} (due to different vertical gravity)*
  - ▷ *H_a is smaller in the ring-like perturbation $\Rightarrow \nu$ is smaller there, too*
  - ▷ *$\dot{M}$ depends on derivative of $\nu \Rightarrow$ it is smaller in inner edge and higher in outer edge of the ring*
  - ▷ *enhanced mass accumulation in the ring $\Rightarrow$ positive feedback*





- rings may work as traps for the dust $\rightarrow$ formation of planets
- rings may decay due to the hydrodynamic instability, if $q > \sqrt{3}$ $(\Omega \sim r^{-q})$

(Papaloizou & Pringle, 1985)

Ring instability - analytical description

- thickness of the dead zone important for $\dot{M}$ (Huré, 2002)
- dimensionless disc thickness: $\delta = \frac{H_a + H_{\rm DZ}}{H_a}$
- rotational velocity corrected to mid-plane pressure:

$$\Omega^2 = \Omega_0^2 + \frac{1}{\rho_m}\frac{\partial P_m}{\partial r}$$

- ring-like perturbation of δ:

$$\delta = \delta_0 + \delta_k \cos(kR), \text{ where } R = r - r_0$$

δ_0 - unperturbed disc thickness, r_0 - position of the ring,

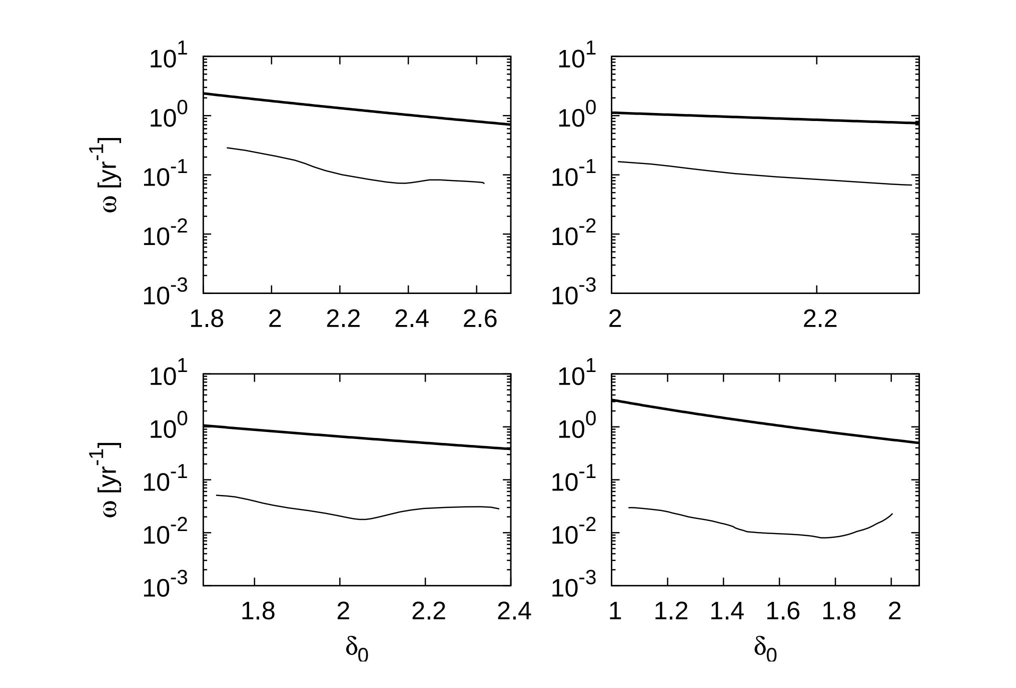
k - wavenumber

- inserting into equations of layered disc, linearization
- $\rightarrow$ perturbation growth rate given by the equation:

$$\dot{\delta_k} = \omega(r_0, \delta_0, k)\delta_k$$

Ring instability - dispersion relation

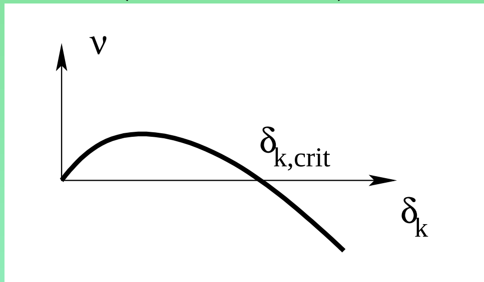
- ω diverges for $k \to \infty$ ($\lambda \to 0$) $\Rightarrow$ the thinnest rings are the most unstable
- radial extent of rings is given by thermal motion of particles $\lambda_0 \sim \frac{c_s}{\Omega}$
- radial extent of rings in simulation in agreement with analytical model ($\sim$ vertical thickness)
- growth rate is smaller in simulation because of the processes that make the radial temperature profile shallower (convection, radiation transport)



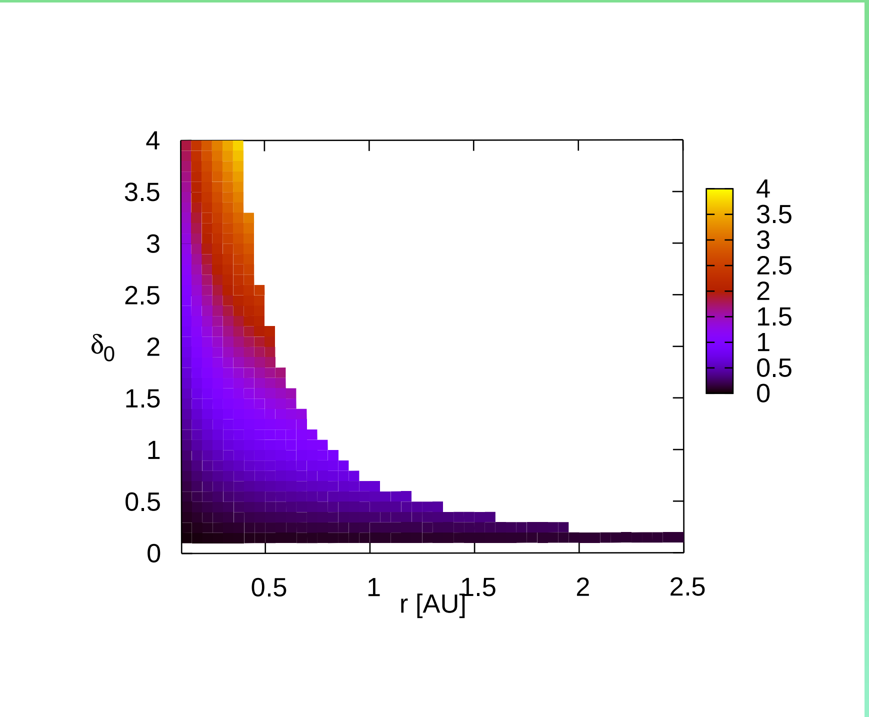
Ring instability - influence of irradiation

analytical approximative approach: $T_i^4 = \frac{3}{8}\tau T_e^4 + W T_\star^4$

- changes the vertical structure of the unperturbed disc
  - *conical disc:* $W = \frac{2}{3\pi}\left(\frac{R_\star}{r}\right)^3$ $\rightarrow$ *viscous term always dominates*
  - *flaring disc:* $W = \alpha_{\rm gr}\left(\frac{R_\star}{r}\right)^2$, *for* $H/r \sim r^{2/7}$
    $\rightarrow$ *irradiation important for* $r > 10$ *AU*
- smoothes the temperature $\Rightarrow$ suppresses the ring instability
  - *grazing angle of the inner edge of the ring:* $\alpha_{\rm gr} \sim \frac{\delta_k}{l(\delta_0+\delta_k)} + \frac{2}{3\pi}\frac{R_\star}{r}$
    $\rightarrow \nu(\delta_k, \delta_0, r, l)$



  - *perturbation of the disc thickness has to reach a certain value, then the instability can grow*

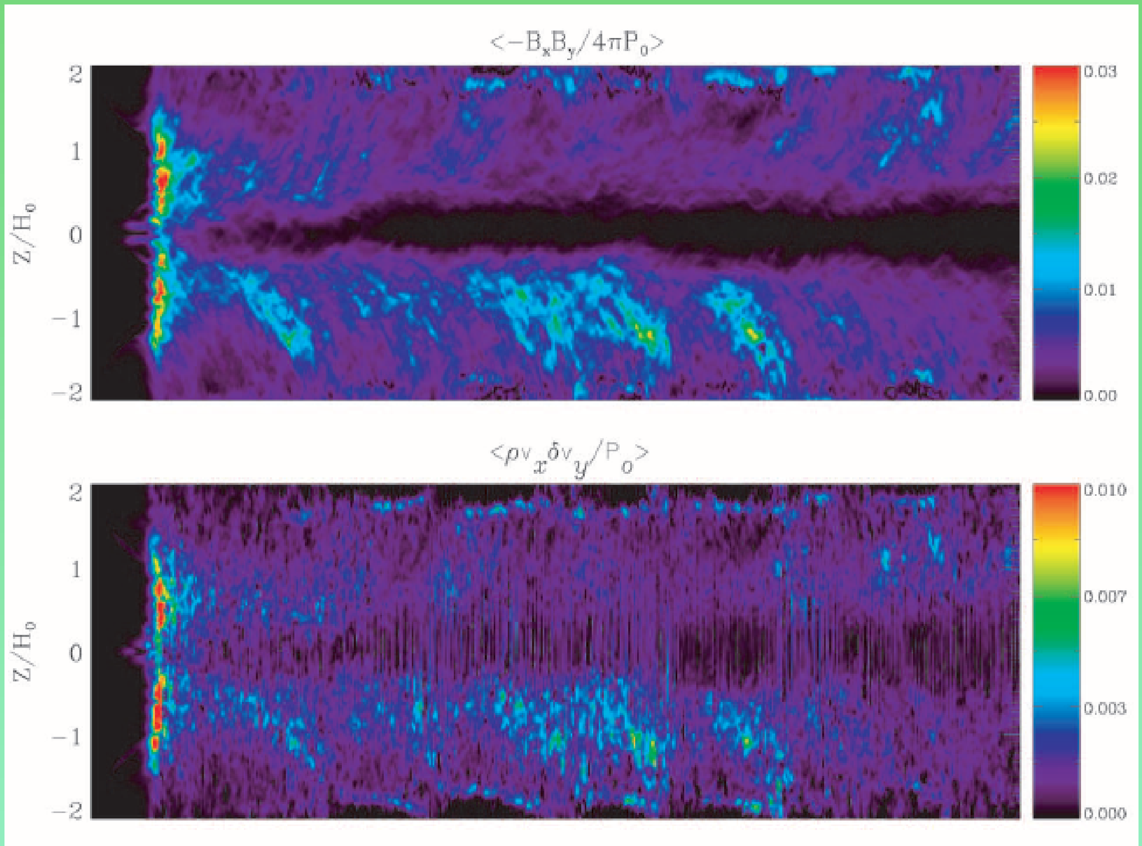


Viscosity in the dead zone - motivation

(Fleming & Stone, 2003)

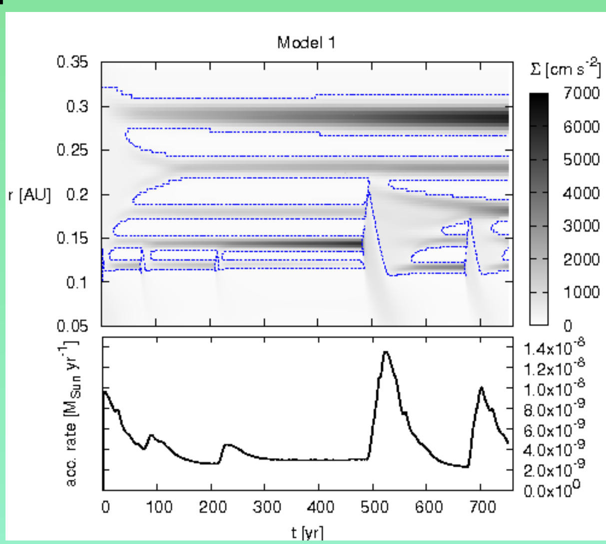
- indications for a small viscosity in the dead zone
- non-ideal MHD ($\eta \neq 0$), shearing box, isothermal EOS

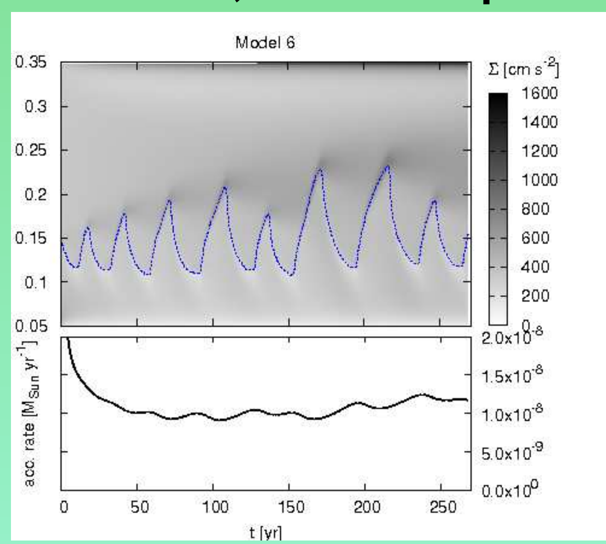
- Maxwell stress vanishes in DZ - MHD turbulence decays
- Reynolds stress non-zero in DZ - HD turbulence survives due to perturbations from active layers (10% of Maxwell s.)

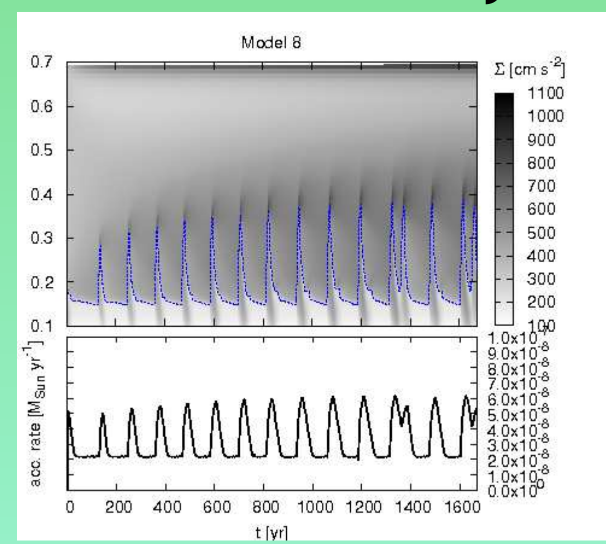


Mini-outbursts - properties, accretion rate

- models with different ν computed for several 1000 orb.:
  - ▷ *definition of ν: $\nu = \alpha c_s H_a$ vs. $\nu = \alpha c_s^2/\Omega$*
  - ▷ *viscosity in active parts $\alpha_a = 0.005, 0.01, 0.02$*
  - ▷ *viscosity in the dead zone $\alpha_{\rm DZ} = 0, 0.01$ and $0.1\alpha_a$*
- rings growth rate cca $10\times$ faster for $\nu = \alpha c_s H_a$
- $\alpha_{\rm DZ} = 0, 0.01\alpha_a \rightarrow$ rings, $\alpha_{\rm DZ} = 0.1\alpha_a \rightarrow$ mini-outbursts
- high viscosity ($\alpha_a = 0.02$) $\rightarrow$ regular mini-outbursts, narrow peaks of the accretion rate, outer part of DZ stationary







Layered disc with $\alpha_{\rm DZ} \neq 0$

- analytical description of layered disc with $\alpha_{\rm DZ} \neq 0$:

$$\dot{M} = 12\pi r^{1/2}\frac{\partial r}{\partial}(\nu_a\Sigma_a + \nu_{\rm DZ}\Sigma_{\rm DZ}), \qquad T_m^4 = \frac{3}{8}\kappa T_e^4\frac{\alpha_a\Sigma_a^2+\alpha_{\rm DZ}\Sigma_{\rm DZ}(\Sigma_{\rm DZ}+2\Sigma_a)}{\alpha_a\Sigma_a+\alpha_{\rm DZ}\Sigma_{\rm DZ}}$$

$$T_e = \frac{9}{4\sigma}\Omega^2(\nu_a\Sigma_a + \nu_{\rm DZ}\Sigma_{\rm DZ})$$

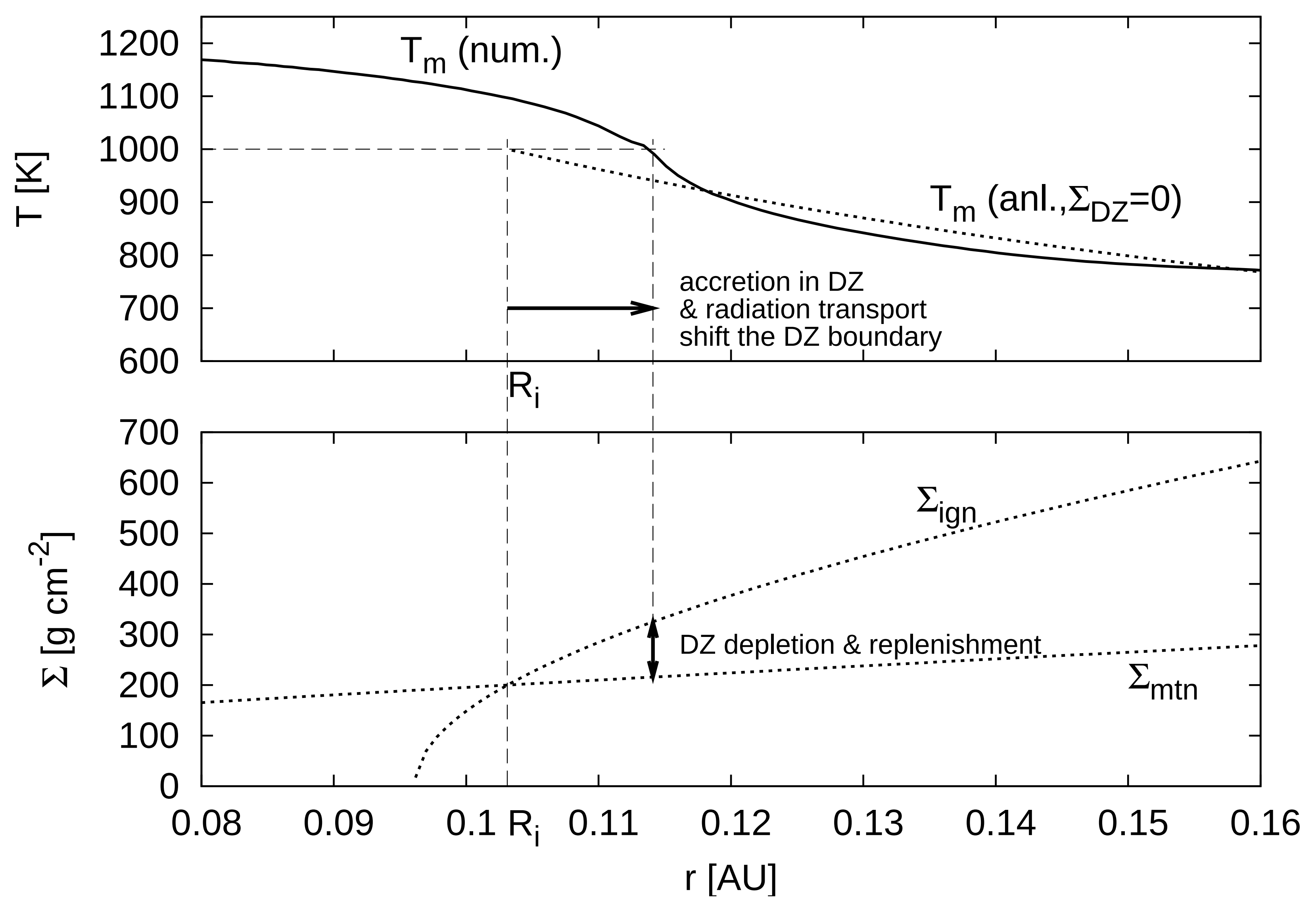
- mid-plane temperature T_m depends on the surface density $\Sigma = 2(\Sigma_a + \Sigma_{\rm DZ})$ (contrary to LD with $\alpha_{\rm DZ} = 0$)
- ignition surface density

$$\Sigma_{\rm ign} = 2\left(\frac{320\sigma}{27}\frac{\mu m_{\rm H}}{k_{\rm B}\Omega\alpha_{\rm DZ}}T_{\rm lim}^{5/2} - \frac{\alpha_a-\alpha_{\rm DZ}}{\alpha_{\rm DZ}}\Sigma_a^2\right)^{1/2}$$

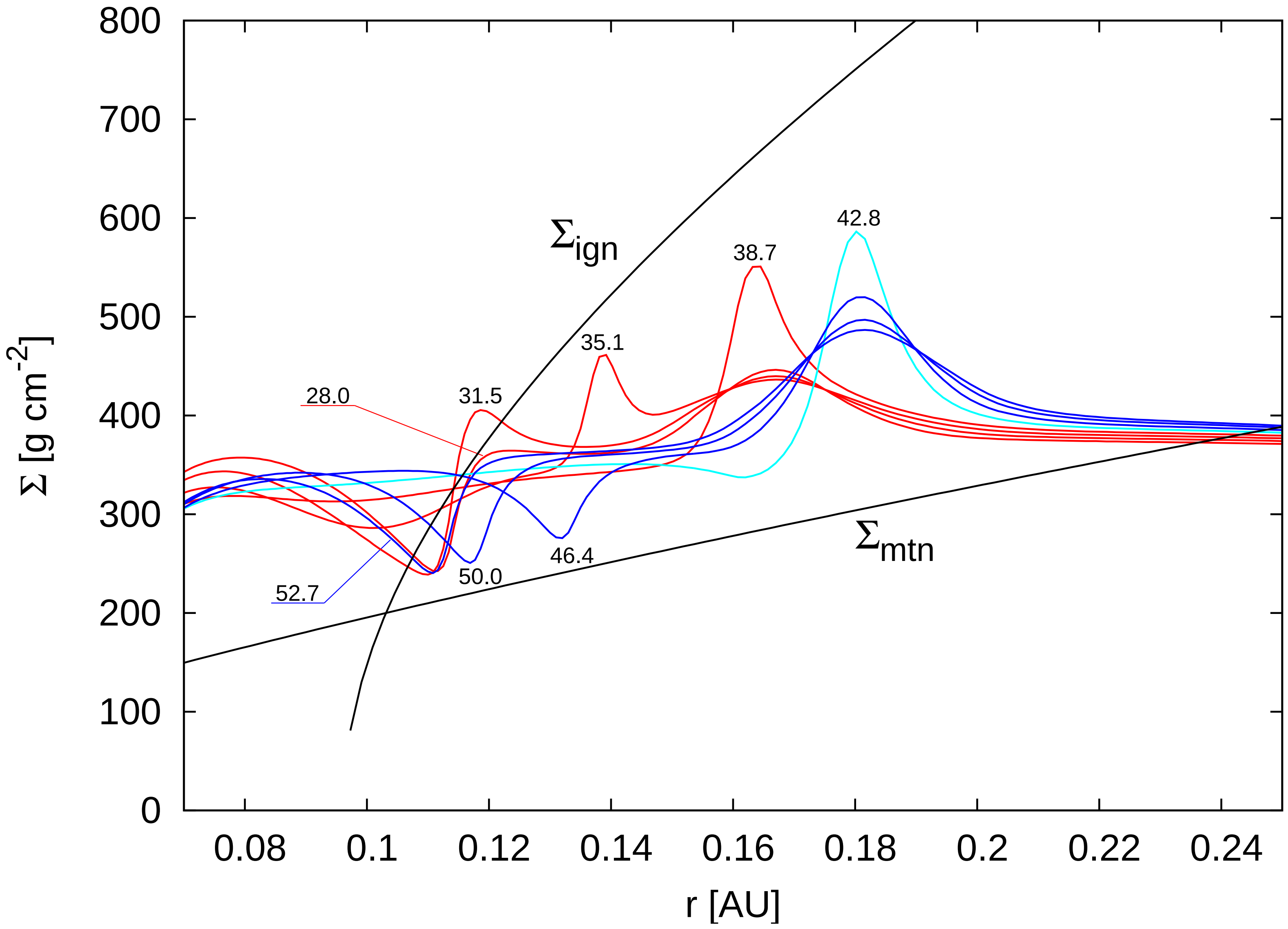
- surface density necessary to maintain the disc active

$$\Sigma_{\rm mtn} = \left(\frac{1280\sigma\mu m_{\rm H}}{27k_B\alpha_a\Omega}\right)^{1/2}T_{\rm lim}^{5/4}$$

Mini-outbursts - mechanism



Evolution of one outburst

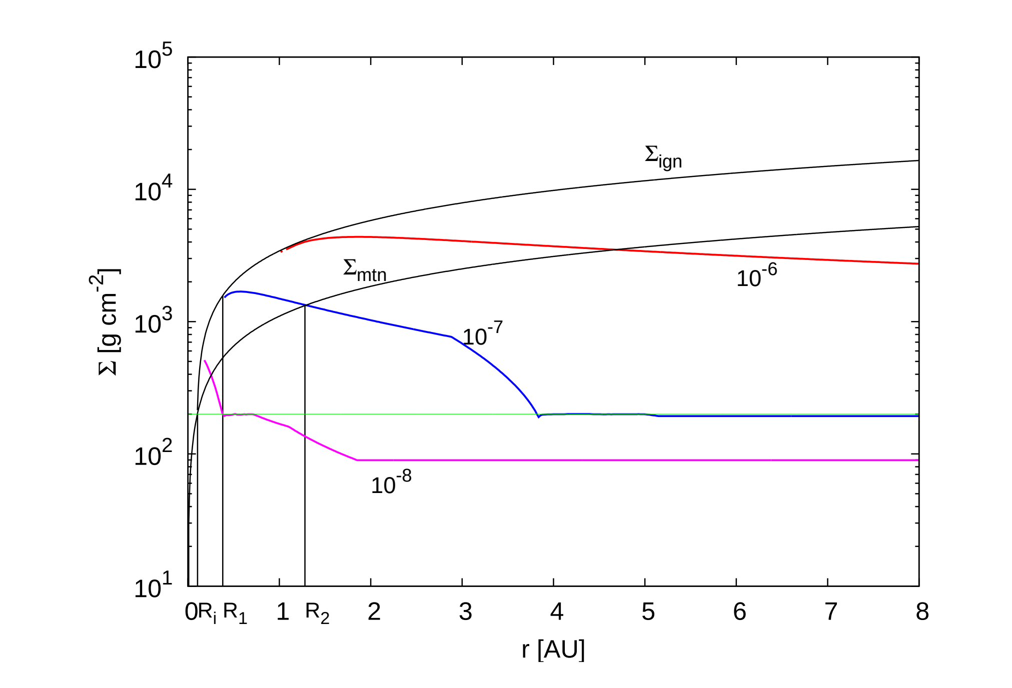


Stationary states

- we search for stationary surface density profiles using 1D code similar to `Stepinski (1999) or Armitage et al. (2001)`
- numerically solve equation $$\dot{\Sigma} = \frac{1}{2\pi r}\frac{\partial \dot{M}}{\partial r}$$
- determines T_m from Σ, decides if LD or αD, computes $\dot{M}$, advects mass between cells

  - ▷ *stationary solution exist above* R_1
  - ▷ *part of DZ between* R_1 *and* R_2 *can be ignited externally*
  - ▷ *it contains mass*

$$2.0\times10^{-7}\mathrm{M}_\odot \text{ for } \dot{M} = 10^{-8}\ \mathrm{M}_\odot\,\mathrm{yr}^{-1}$$
$$2.7\times10^{-4}\mathrm{M}_\odot \text{ for } \dot{M} = 10^{-7}\ \mathrm{M}_\odot\,\mathrm{yr}^{-1}$$
$$0.01\mathrm{M}_\odot \text{ for } \dot{M} = 10^{-6}\ \mathrm{M}_\odot\,\mathrm{yr}^{-1}$$



Conclusions

- the dead zone with zero viscosity is a subject of the ring instability
- the instability occurs since the local enhancement of the disc thickness leads to a drop of the viscosity and the accretion rate profile which increases the original disc thickness enhancement

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- the dead zone with zero viscosity is a subject of the ring instability
- the instability occurs since the local enhancement of the disc thickness leads to a drop of the viscosity and the accretion rate profile which increases the original disc thickness enhancement
- with $\alpha_{\rm DZ} \neq 0$ the oscillations of the inner edge of DZ occur - continuously remove mass from the inner part of DZ
- a stationary accretion ($\dot{M} = \rm const$) may occur at higher radii; part of this area is combustible - can be made active and part of its mass accreted in an outburst type event

Future

- going to 3D (to see what happens with rings and mini-outbursts without the assumption of spherical symmetry)
- irradiation by the central star
- more complicated ionization structure (ionization by X-rays, . . .)

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