Hydrodynamic equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes (\rho \mathbf{u})) + \nabla p = 0$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{u}(E + p)) = 0,$$

- conservation laws
- can be obtained from statistical physics, but firstly obtained fenomenologically
- generate chaos $\rightarrow$ turbulence
- solution may break $\rightarrow$ discontinuity (shock-wave) appears

Wave steepening -> Shock wave

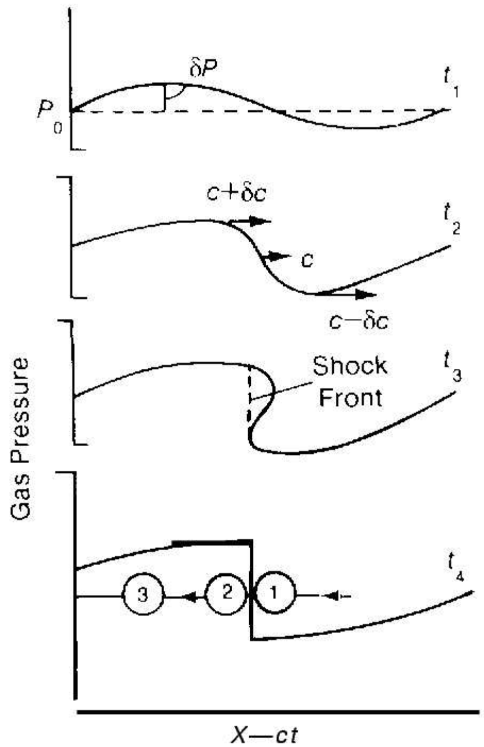
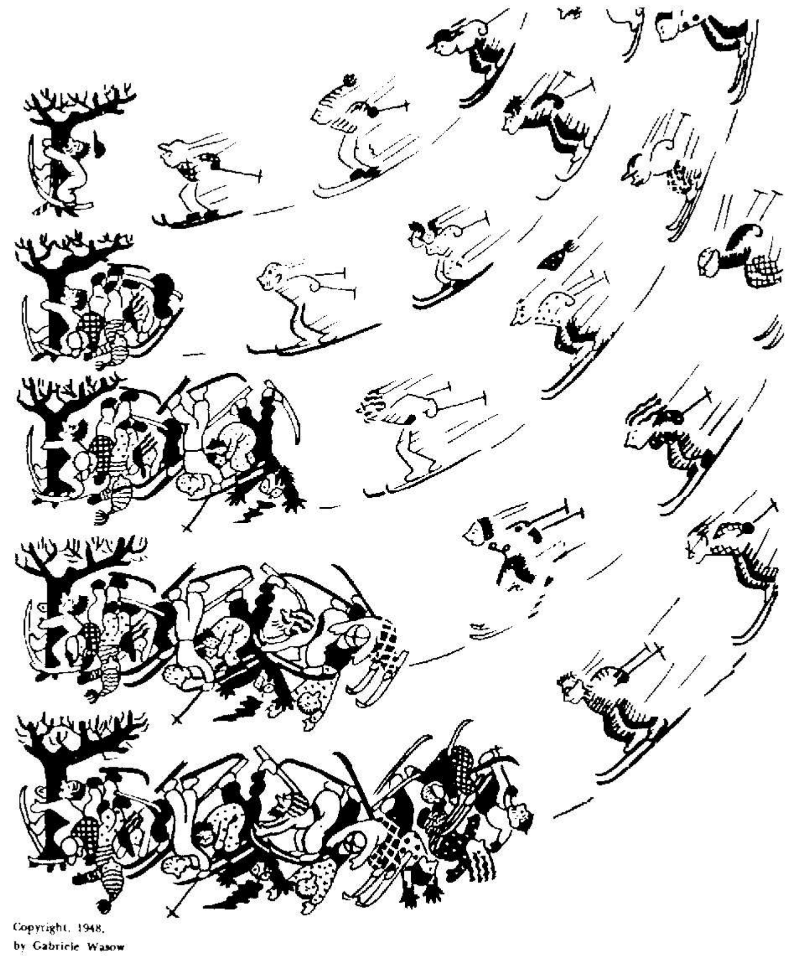


Fig. A. Self-steepening of a finite-amplitude sound wave. In the region where the state variables of the wave (here, pressure) would become multi-valued, irreversible processes dominate to create a steep, single-valued shock front (vertical dashed line).



An example of a receding shock wave. From **Supersonic Flow and Shock Waves** *by R. Courant and K. O. Friedrichs (New York:Interscience Publishers, Inc., 1948),*

Irreversibility

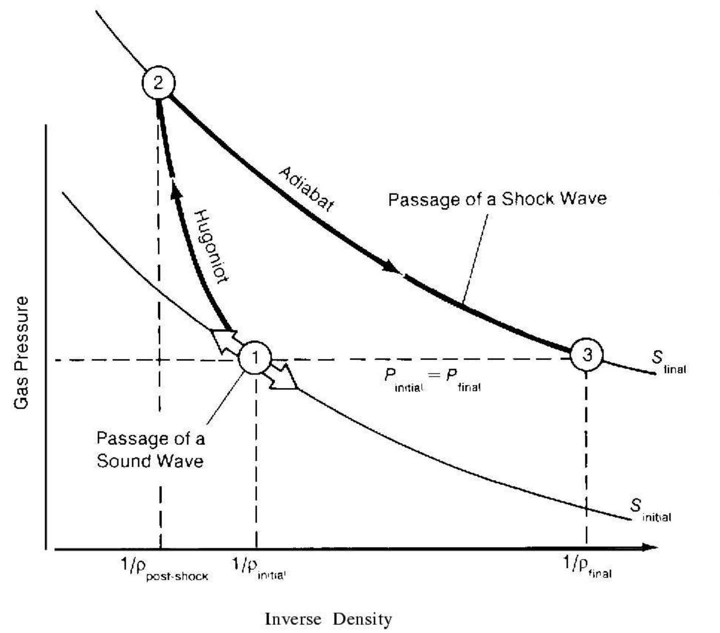
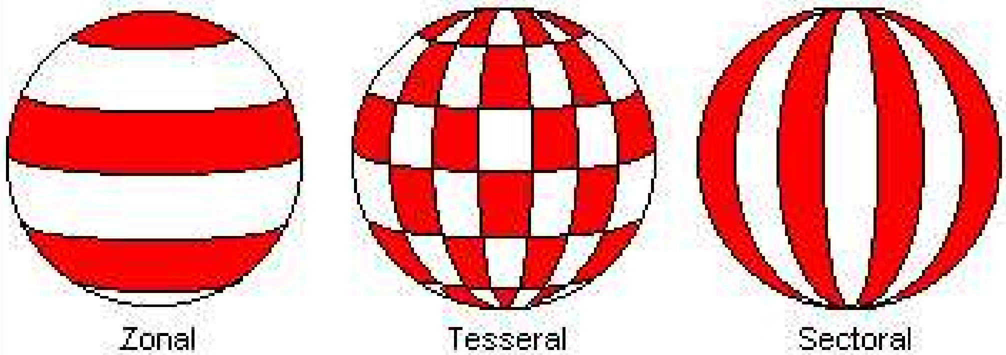


Fig. B, Effects of the passage of a sound wave and of a shock wave. As a sound wave passes through a gas, the pressure and density of the gas oscillates back and forth along an adiabat (a line of constant entropy], which is a reversible path. In contrast, the passage of a shock front causes the state of the gas to jump along an irreversible path from point 1 to point 2, that is, to a higher pressure, density, and entropy. The curve connecting these two points is called a Hugoniot, for it was Hugoniot (and simultaneously Rankine) who derived, from the conservation laws, the jump conditions for the state variables across a shock front, After passage of the shock, the gas relaxes back to point 3 along an adiabat, returning to its original pressure but to a higher temperature and entropy and a lower density. The shock has caused an irreversible change in the gas.

Spherical harmonics



Types of Spherical Harmonics

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \phi}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 \phi}{\partial \psi^2} = 0$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Theta}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 \Theta}{\partial \psi^2} + n(n+1)\Theta = 0$$

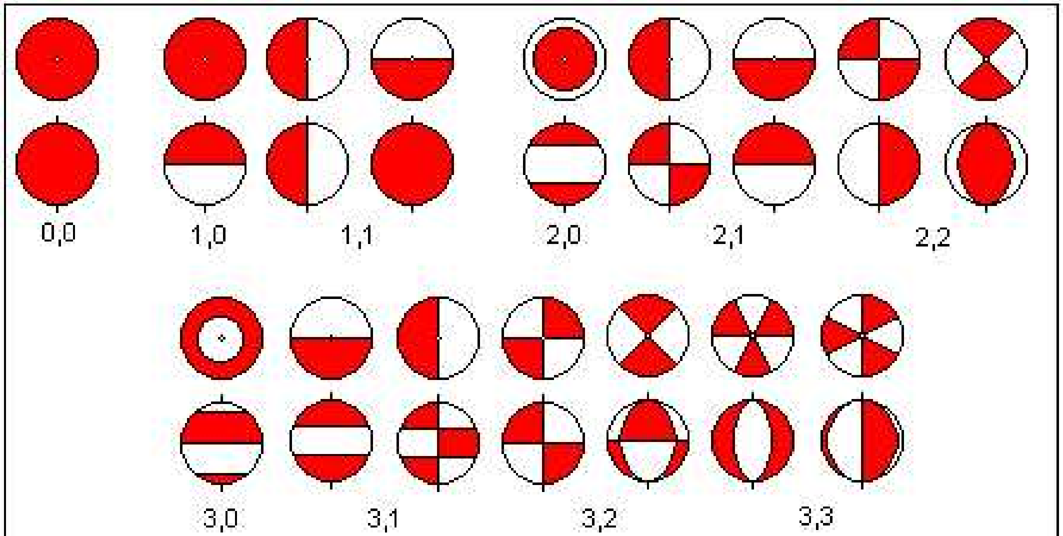
$$\phi = \left\{ \begin{matrix} r^n \\ \frac{1}{r^{n+1}} \end{matrix} \right\} P_n^m(\cos\theta)\, e^{im\psi}$$

$n = 0, 1, \ldots \quad m = -n, -n+1, \ldots, n-1, n$

$$P_n^m(u) = (1 - u^2)^{\frac{m}{2}} \frac{d^m}{du^m} P_n(u)$$ associated Legendre functions

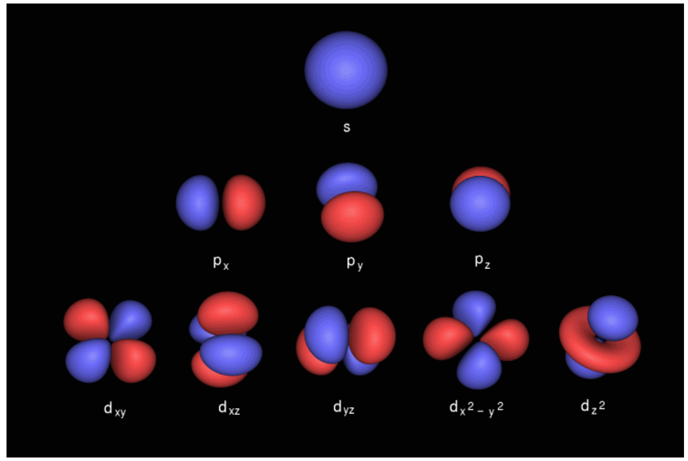
$$P_n(u) = \frac{1}{2^n n!} \frac{d^n}{du^n} (u^2 - 1)^n$$ Legendre polynomials

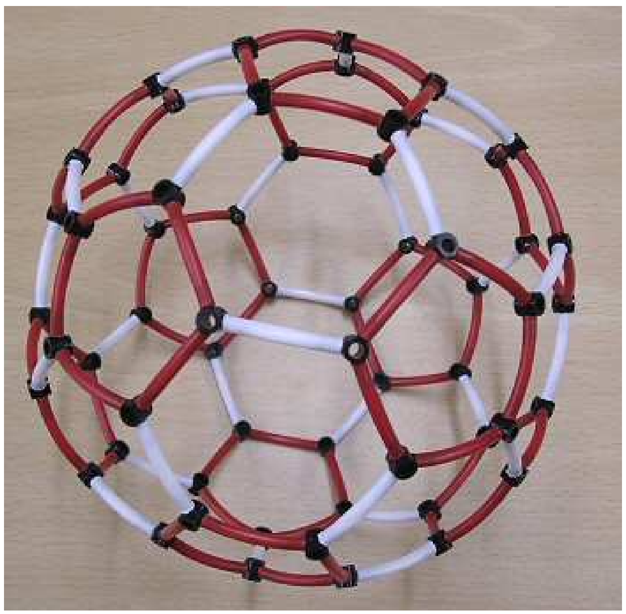
Spherical Harmonics

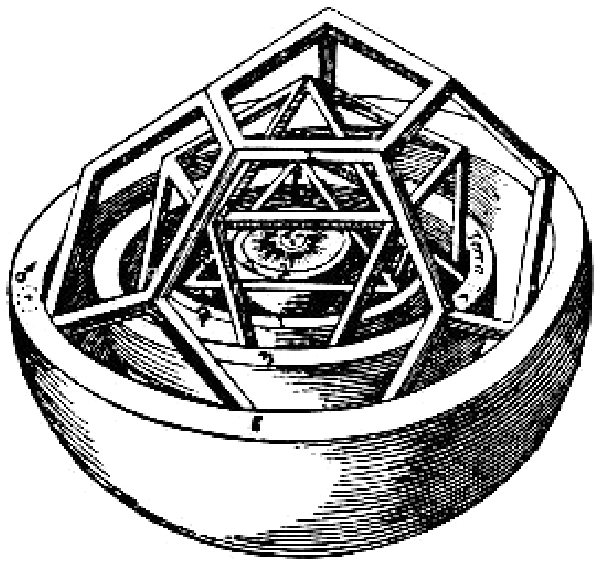


Spherical Harmonics, Nodal Surfaces n = 0,1,2,3

Spherical harmonics vs. beauty







References