Gravitational fragmentation of expanding shells

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Outline:

1. Expanding shells in astrophysics

2. New tree gravity solver for FLASH
   - *algorithm*
   - *benchmarks*

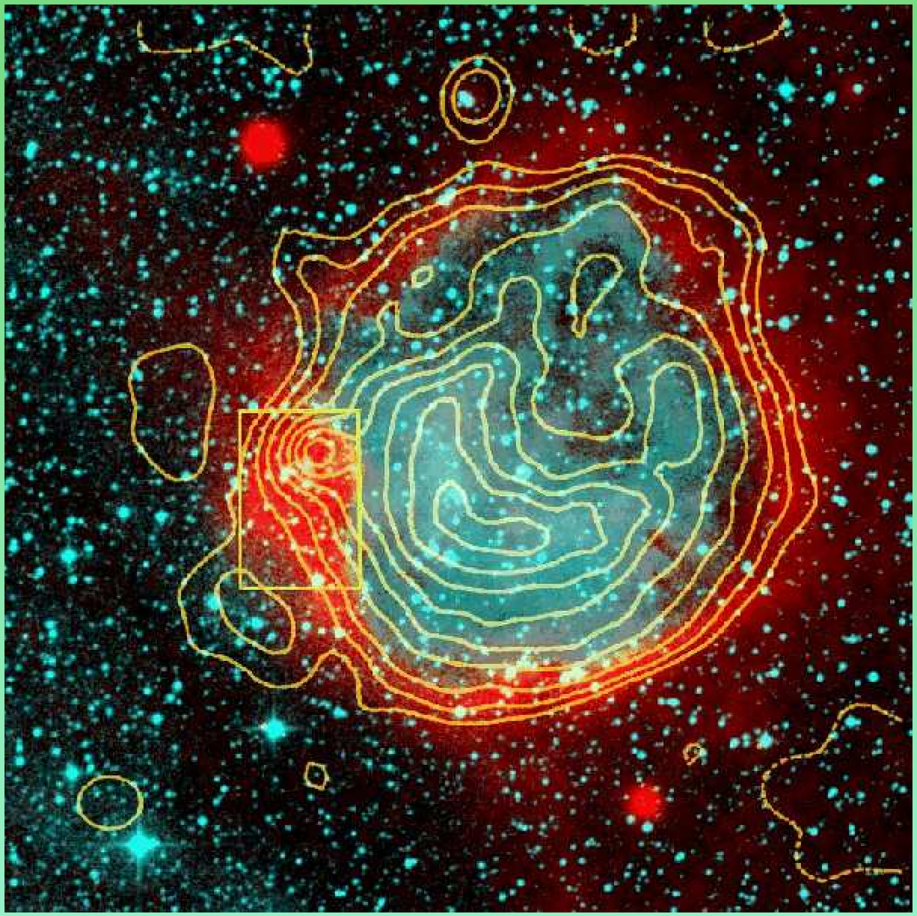
3. Momentum driven shell
   - *justification of the (over-)simplified model*
   - *simulation setup, analysis*

4. Gravitational instability
   - *thin-shell dispersion relation*
   - *AMR vs. SPH vs. thin-shell approx.*
   - *GI of the thick shell?*

Motivation 1: Collect and collapse

- C&C (Elmegreen & Lada, 1977): SF at peripheries of HII region
- gravitation instability of material accumulated between IF and SF
- massive stars can be formed → self-propagating SF
- HII region Sh 104

  (Deharveng et al., 2003)
  - ▷ *contours: thermal radio continuum (1.46 GHz)*
  - ▷ *red: mid-IR emission (dust - PAHs)*
  - ▷ *turquoise: ionized gas*

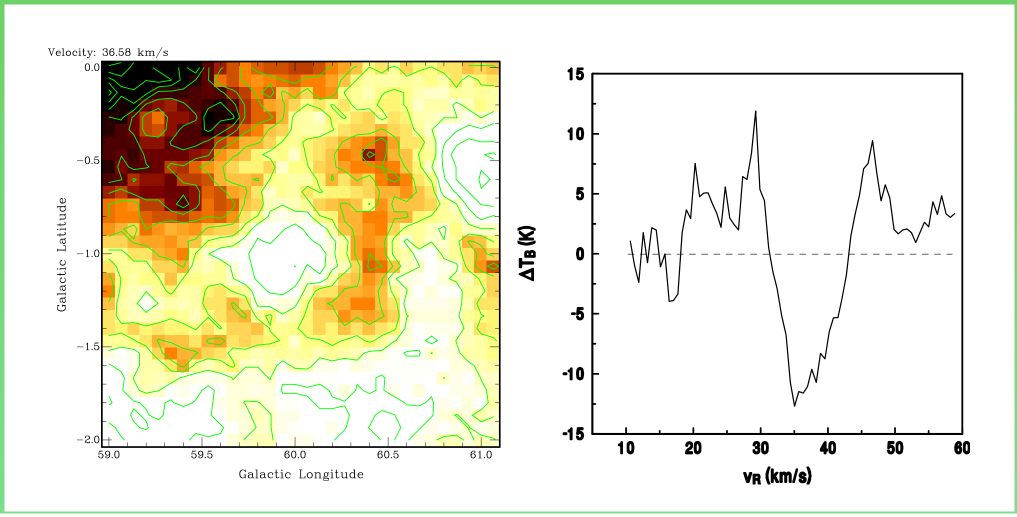


- UC HII region in the dust ring → exciting embedded cluster
- 17 C&C candidate regions suggested by

  Deharveng (2005)

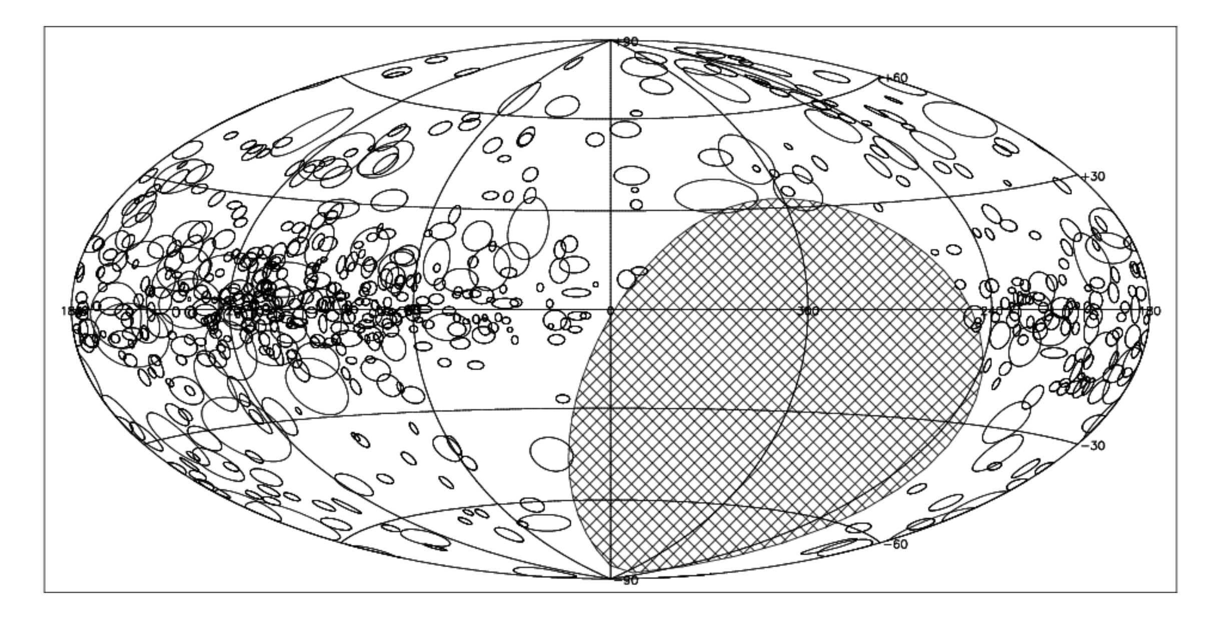
Motivation 2: HI shells and supershells

- typically larger structures (10pc - 1kpc) formed by OB associations, GRB, encounters, turbulence
- formation of GMC



- 300 shells identified in Leiden-Dwingeloo survey

(Ehlerová et al. 2005)



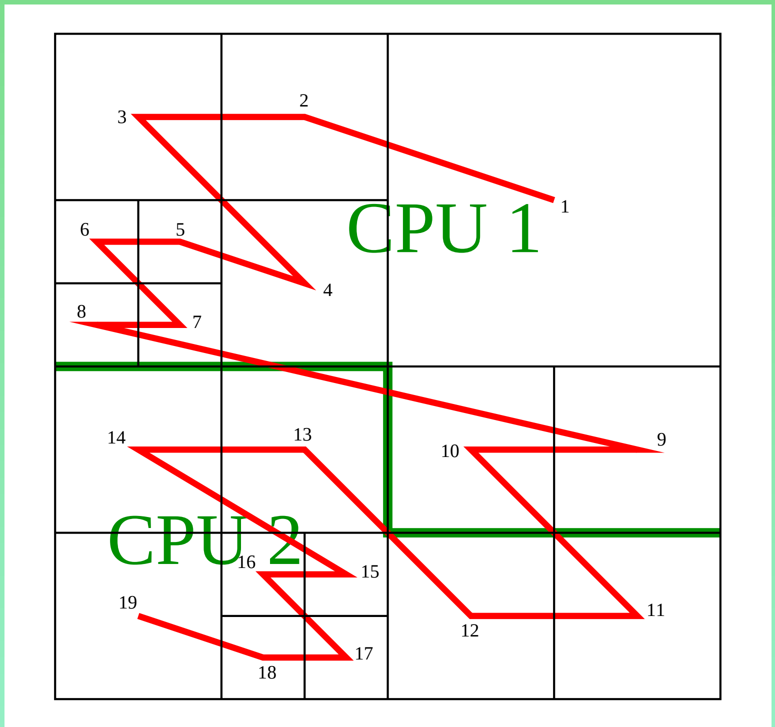
Tree gravity solver for FLASH

- Why new gravity solver?

Default multi-grid solver:

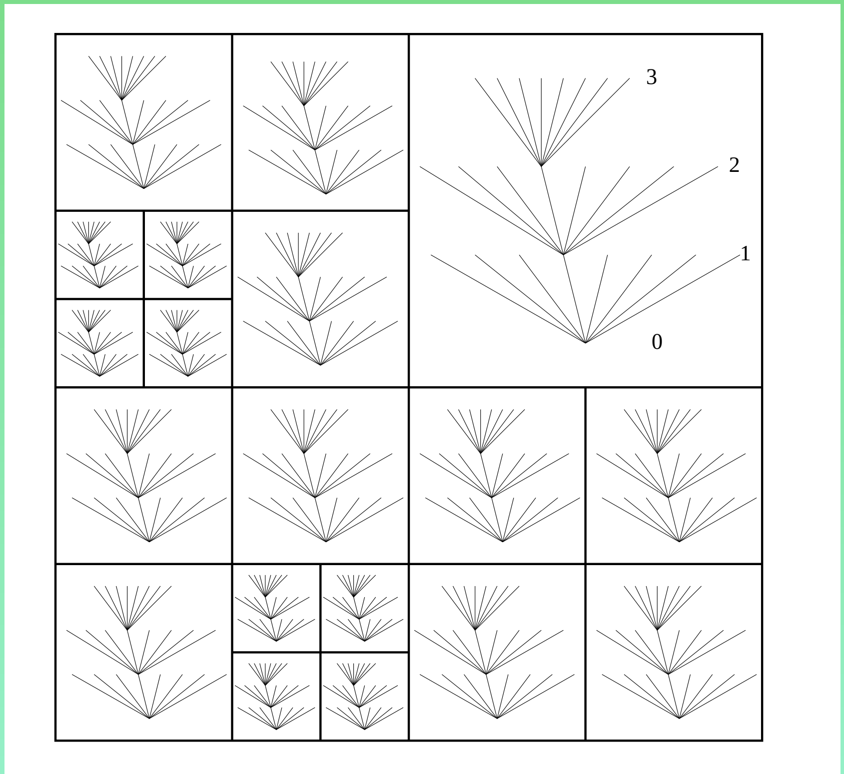
- ▷ *scales bad on slow networks (high communication requirements)*
- ▷ *consumes a lot of memory:*
  multi-grid (1500 blocks) . . . 1836 MB
  tree (1500 blocks) . . . 1320 MB
  tree (2100 blocks) . . . 1838 MB
- ▷ *iterative solver is not ideal if most of mass moves quickly with respect to the grid*

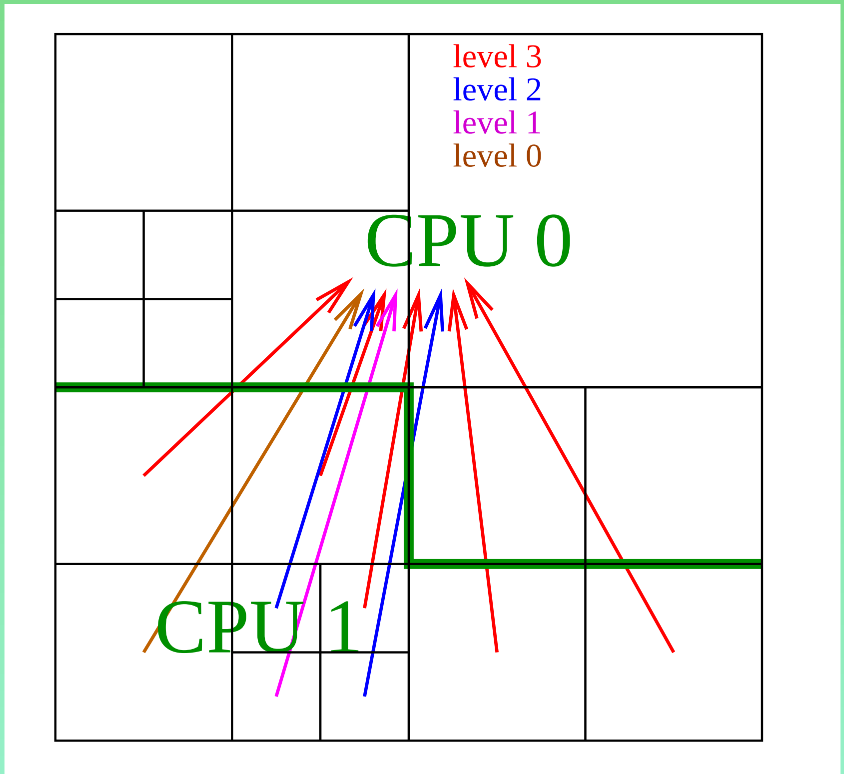
- Why the tree code?
  - ▷ *FLASH AMR based on the octal tree*
  - ▷ *good experience from SPH codes*
  - ▷ *no communication between neighbour cells (4 layers of ghost zones ⇒ a lot of RAM and bandwidth needed)*



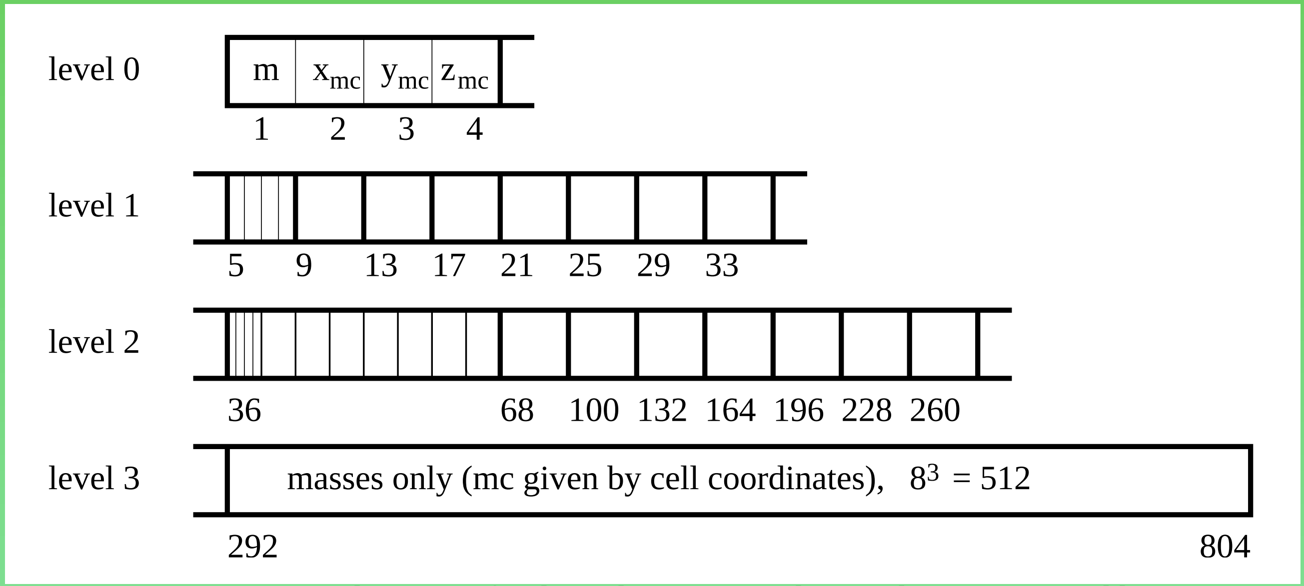
Gravity tree

- octal tree in each block + global tree of AMR blocks
- only masses and positions of mass centres (no quadrapole moments)
- communication:
  - ▷ *1. the global tree distributed among the all processors*
  - ▷ *2. individual block trees: sent only down to a necessary level*





Tree in RAM



L . . . number of the lowest level (typically 3)

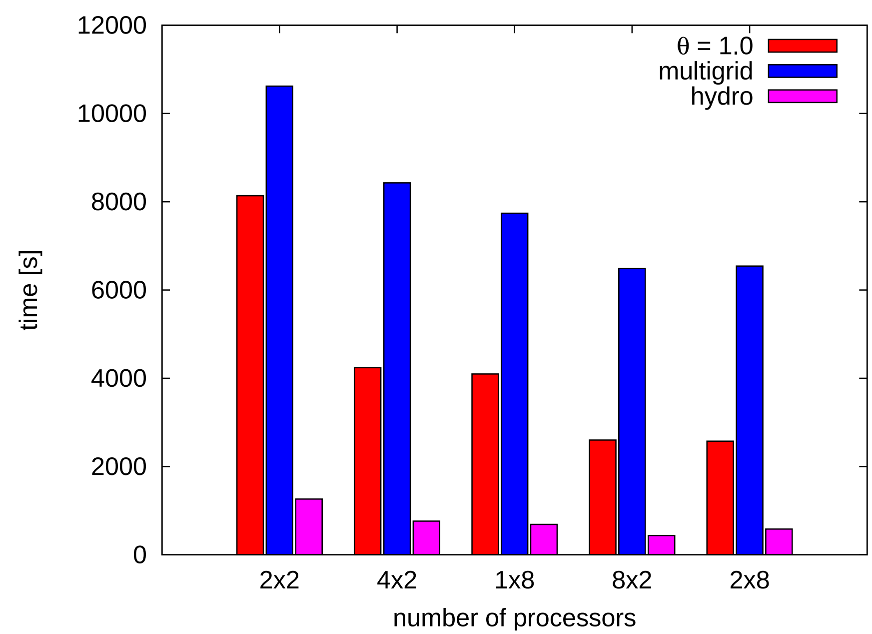
$$\text{Tree size} = 8^L + 4\sum_{i=0}^{L-1} 8^i = 8^L + 4\frac{8^L - 1}{7}$$

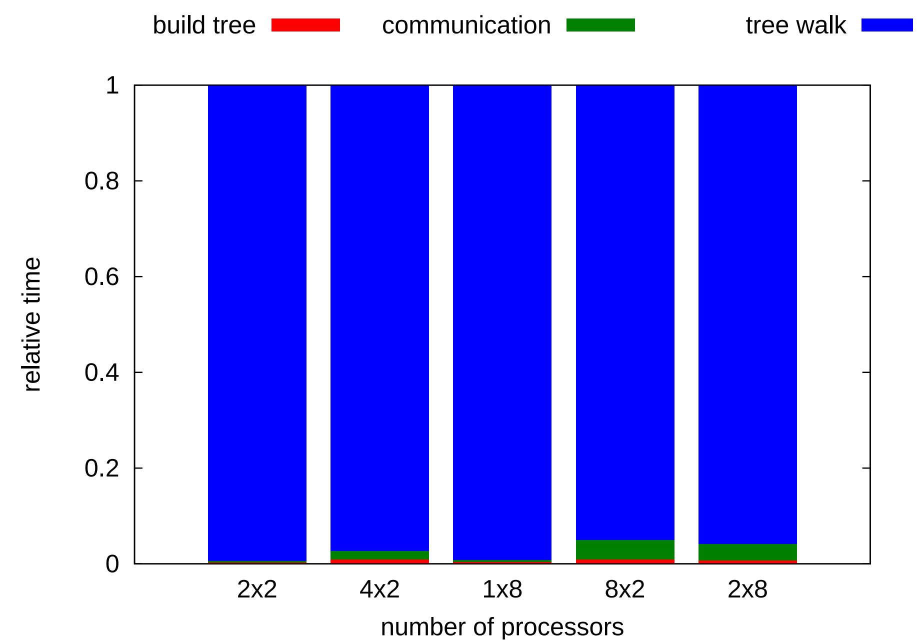
tree nodes identified by multi-index - integer array of size L: (l_1, l_2, l_3); $l_i =$

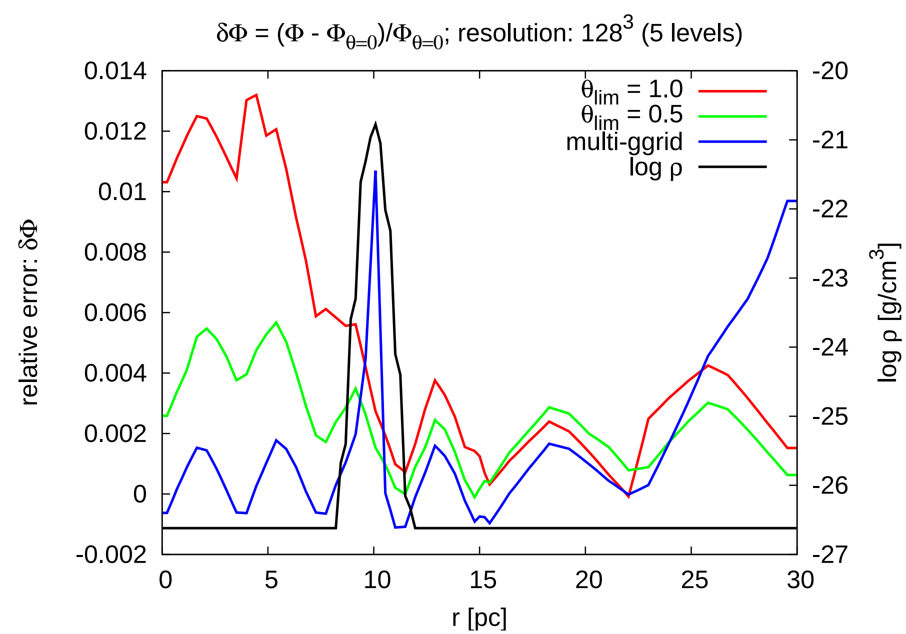
- ▷ *1-8 . . . number of node on i-th level*
- ▷ *0 . . . multi-index (i.e. node) is of level i-1*

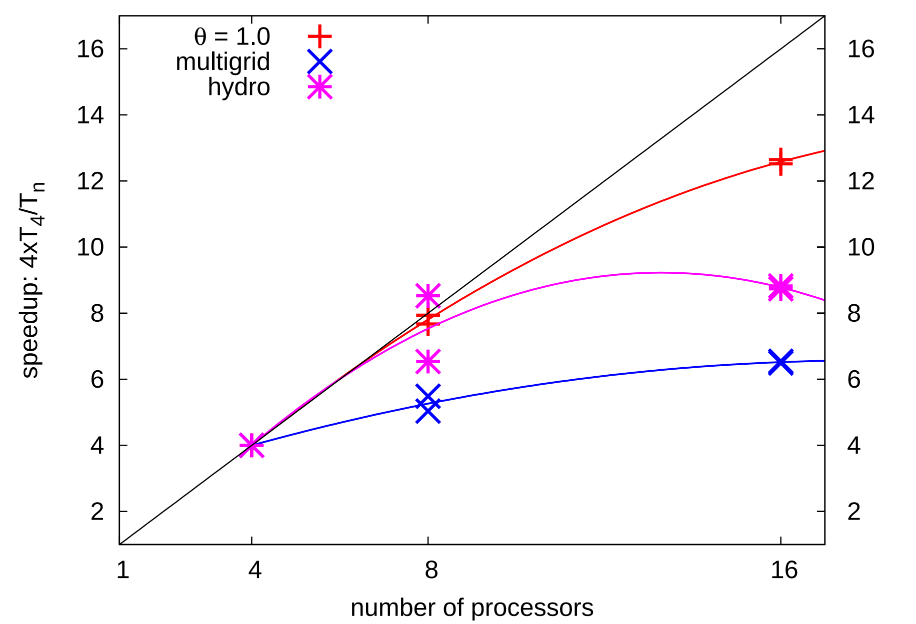
- tree walk on individual processors 100% parallel

Benchmarks (2GHz Opterons, ethernet)

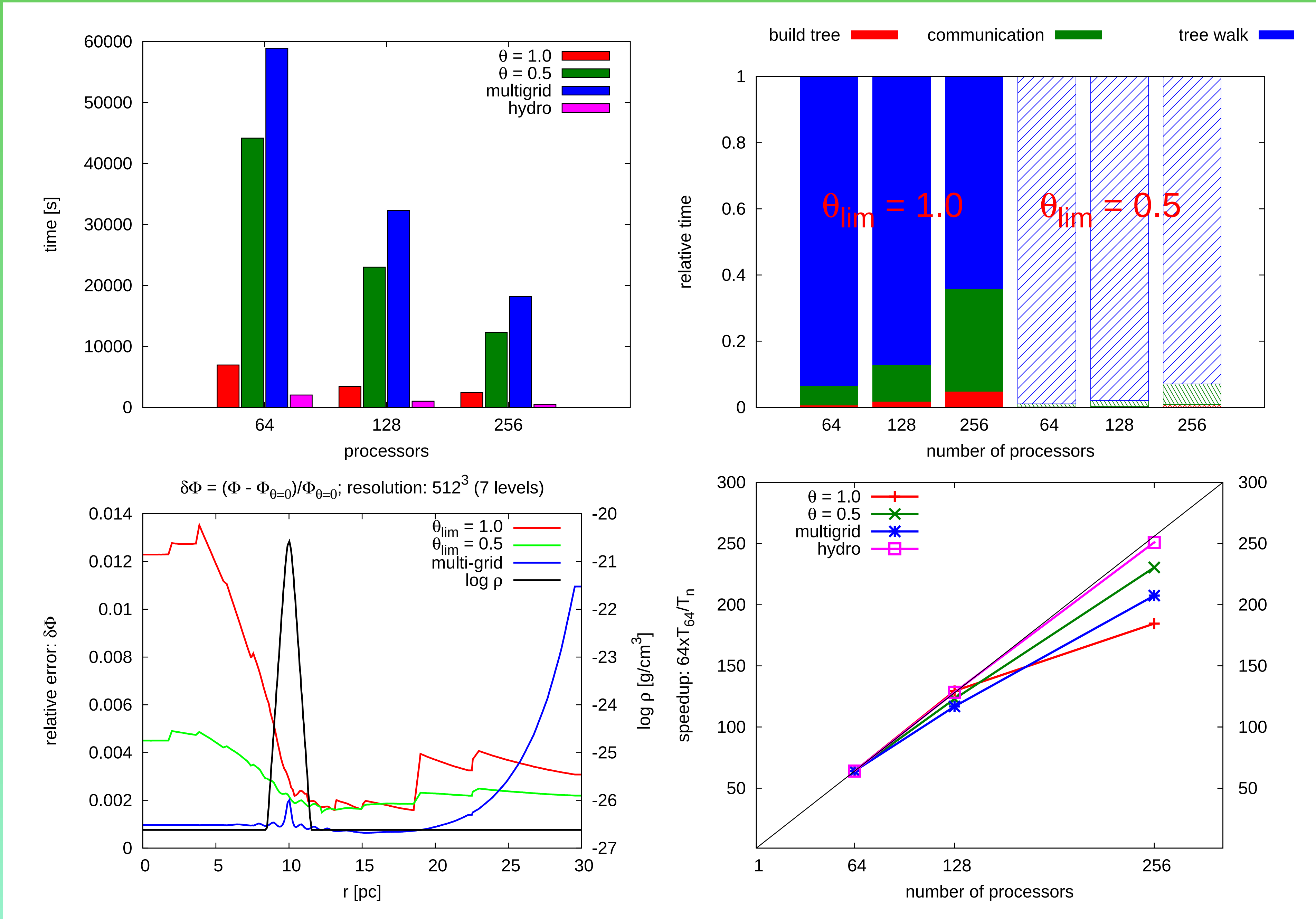




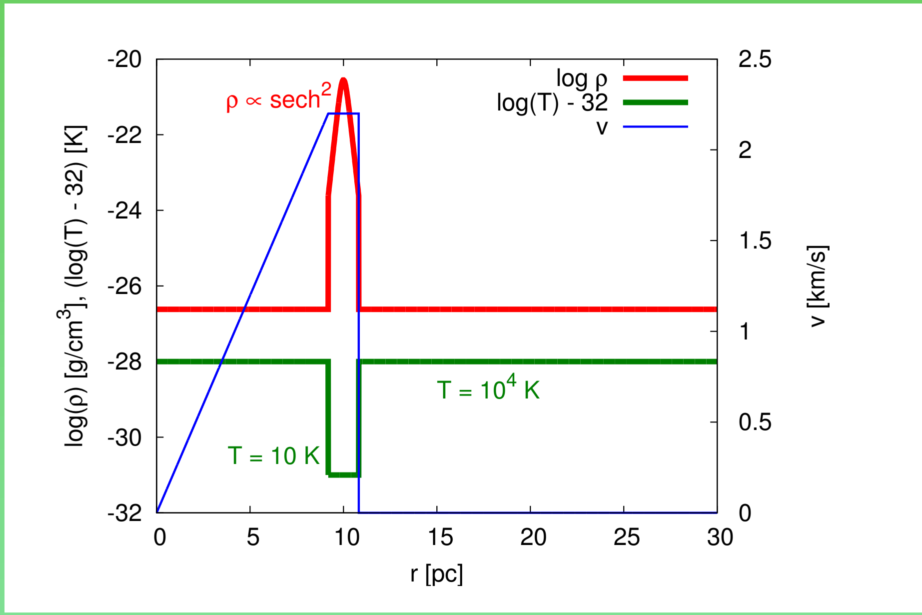




Benchmarks (3GHz Xeons, Infiniband)

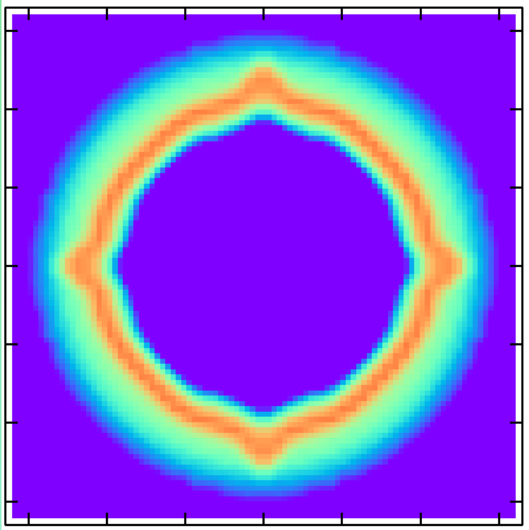


Momentum driven (ballistic) shell

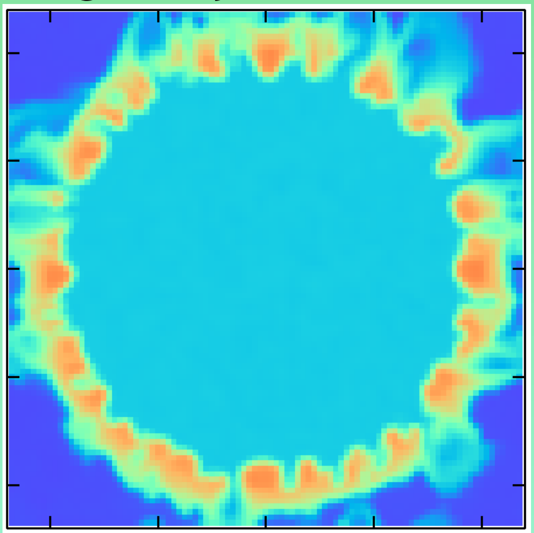


- extremely simplified model to avoid instabilities other than the gravitational one

accretion of ambient gas
$\longrightarrow$ ram pressure vs. thermal pressure
$\longrightarrow$ Vishniac (1983) instability:



non-zero effective radial gravitational force
$\longrightarrow$ Rayleigh-Taylor instability:



Gravitation instability of the thin shell

- GI in the expanding accreting shell studied analytically by Elmegreen (1994), Vishniac (1983) and Whitworth et al. (1994)

$$\omega(l) = -\frac{3V}{2R} + \sqrt{\frac{V^2}{4R^2} + \frac{GMl}{2R^3} - \frac{c_s^2 l^2}{R^2}}$$

- linearised perturbed 2D HD eqs in the shell:

pressure, gravity, stretching, accretion

$$\Sigma_0 R \frac{\partial \mathbf{\Omega}}{\partial t} = -c_s^2 \nabla \Sigma_1 + \Sigma_0 \nabla \Phi_1 - \Sigma_0 \mathbf{\Omega} V - \cancel{3\Sigma_0 \mathbf{\Omega} V}$$

$$\frac{\partial \Sigma_1}{\partial t} = -\Sigma_0 R \nabla_{\mathrm{T}} \cdot \mathbf{\Omega} - 2\Sigma_1 \frac{V}{R}$$

stretching

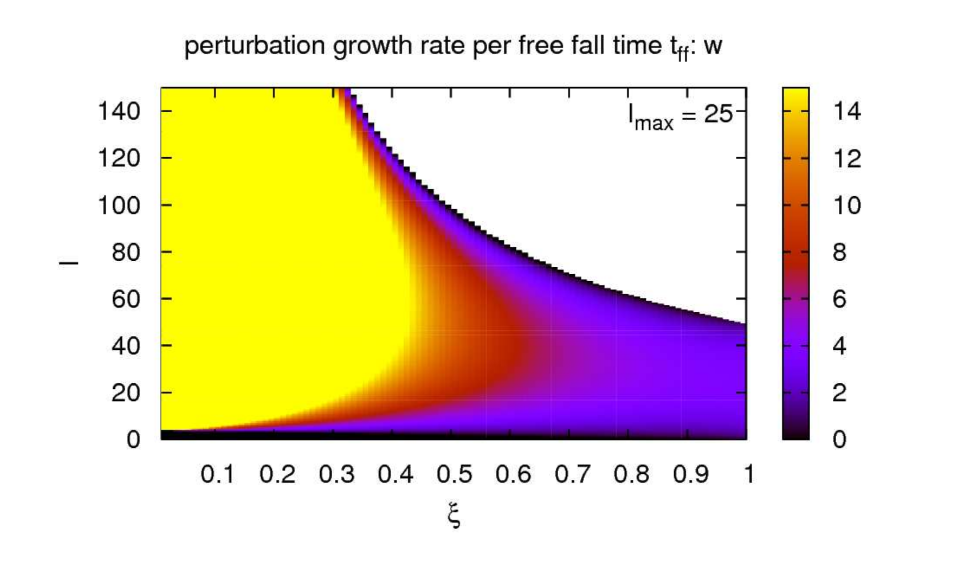
$$\nabla^2 \Phi_1 = 4\pi G \Sigma_1 \delta(r - R)$$

Dispersion relation

- dimensionless formulation:

$$\xi = R/R_{\rm max}, \quad t_{\rm ff} = \frac{\pi R_{\rm max}^{3/2}}{2(GM)^{1/2}}, \quad l_{\rm max} = \frac{GM}{4c_s^2 R_{\rm max}}$$

- three params (V_0, M, c_s) reduced into one: $l_{\rm max}$



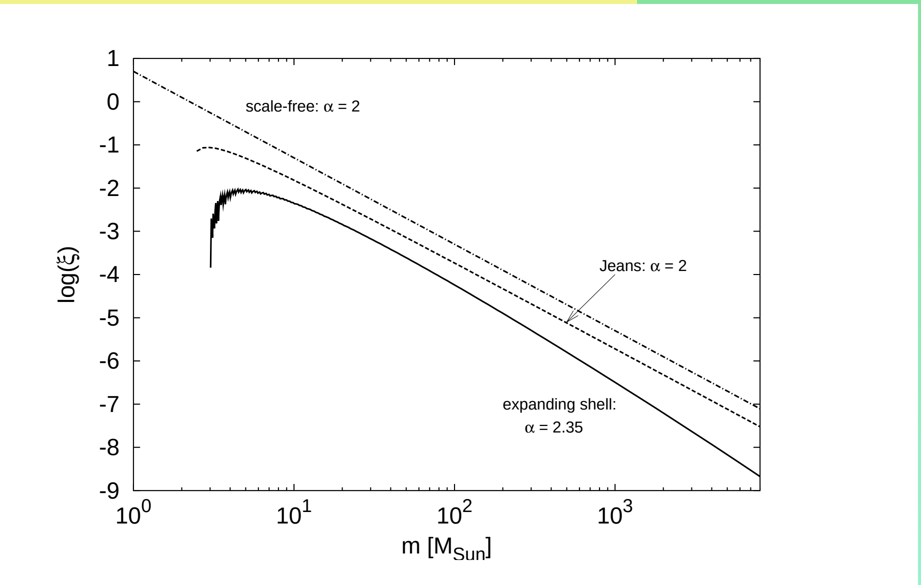
$$\begin{aligned} w(l, \xi, l_{\rm max}) &= \omega(l, R, V, \Sigma, c_s) t_{\rm ff} \\ &= -\frac{3\pi}{4}(1-\xi)^{1/2}\xi^{-3/2} + \frac{\pi}{4}\left[\xi^{-3}(1+2l) - \xi^{-2}\left(1 + \frac{l^2}{l_{\rm max}}\right)\right]^{1/2} \end{aligned}$$

- mass spectrum of fragments:

$$dN \sim \left(\int \omega(l,t)dt\right) \times l^2 dl$$

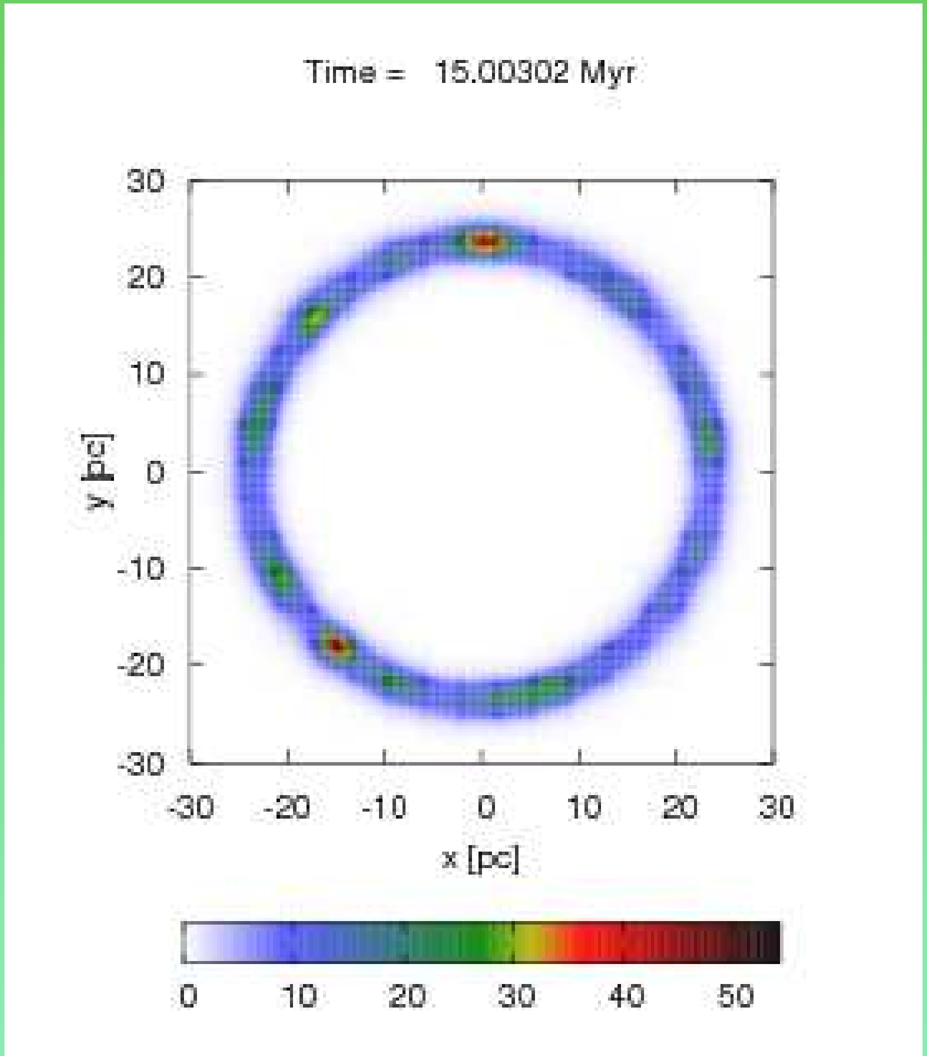
$$l \rightarrow m\text{: } m = \pi(\pi l/R)^2 \Sigma$$

$$dN \sim f_{\rm IMF}(m)dm$$



Simulation setup

- parameters:
  - ▷ $M = 2 \times 10^4$ *M*$_\odot$
  - ▷ $T = 10$ *K*
  - ▷ $R_0 = 10$ *pc*
  - ▷ $V_0 = 2.2$ *km/s*
  - ▷ $R_{\rm max} = 22.9$ *pc*
  - ▷ $l_{\rm max} = 22.6$
  - ▷ $t_{\rm ff} = 18.1$ *Myr*



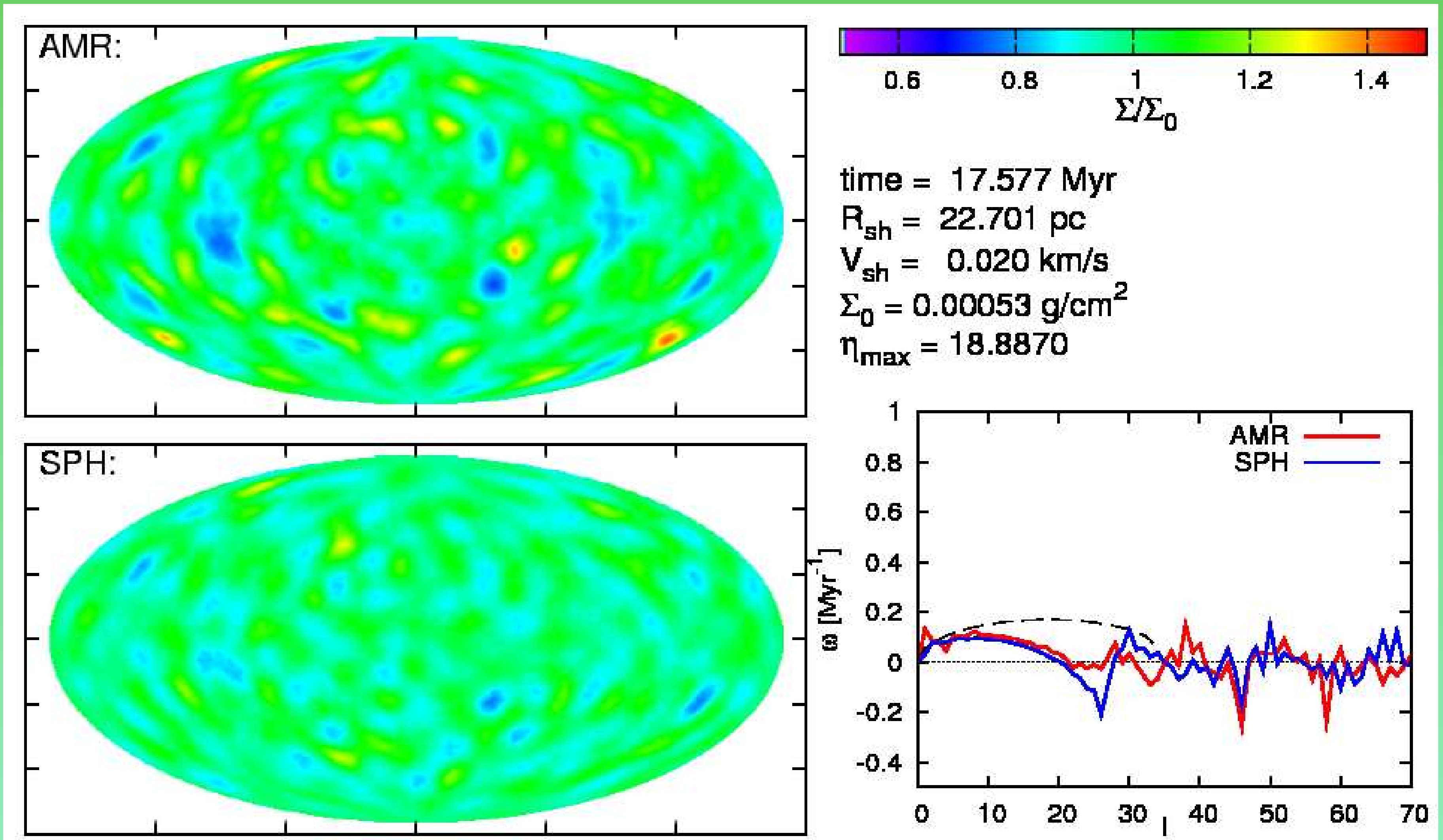
- external pressure: $P_{\rm ext}$
  - ▷ $P_{\rm ext} = 10^{-15}$ *dyne cm*$^{-3}$ *(low)*
  - ▷ $P_{\rm ext} = 10^{-13}$ *dyne cm*$^{-3}$ *(high)*

- initial conditions (ρ, v pert.):
  - ▷ *monochromatic (spherical harm)*
  - ▷ *random vel. field with Maxwell distr. (remapping: SPH → AMR)*

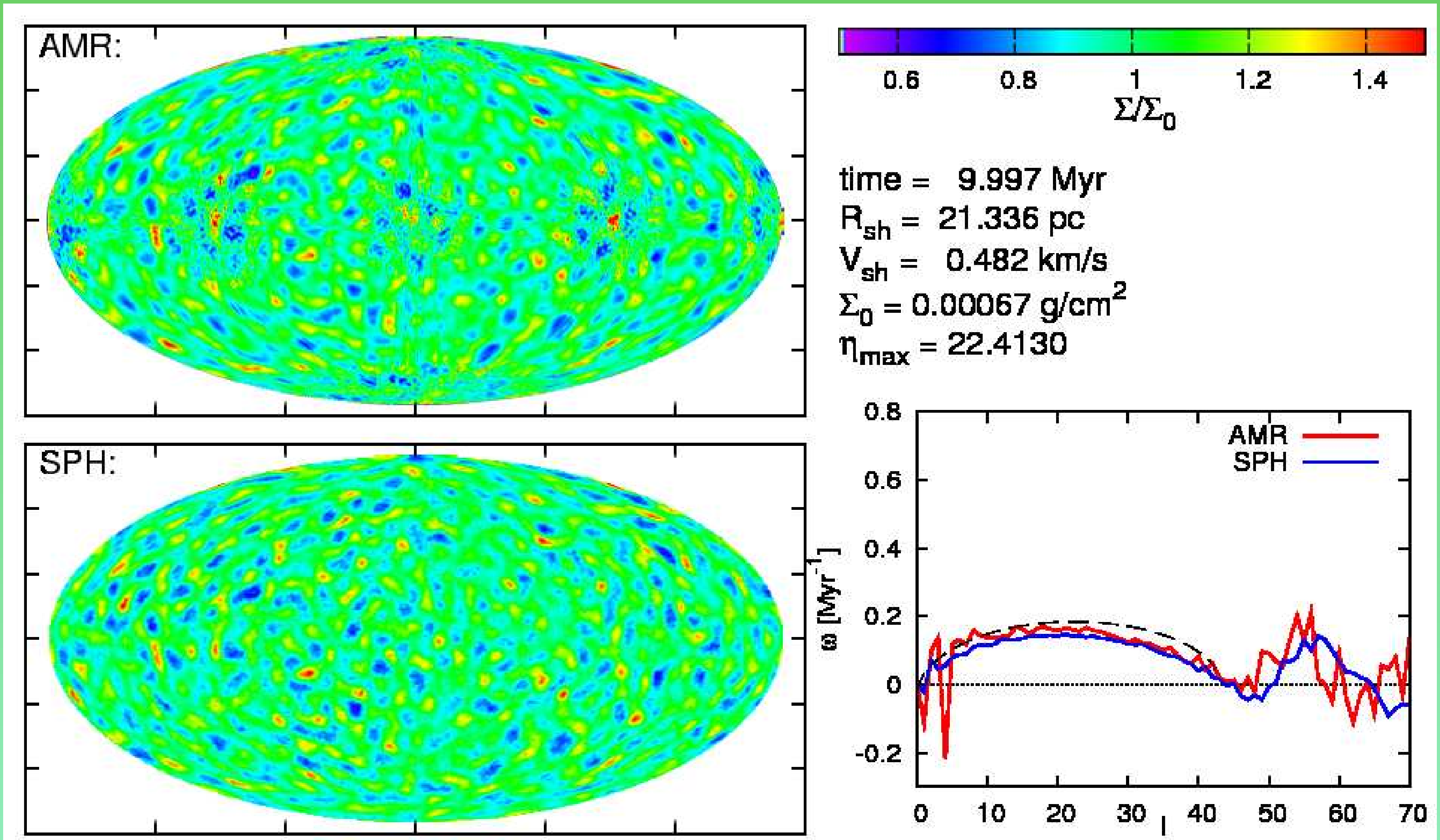
- decomposition into sph. harm., power spectrum: C_l

$$\omega(l) = \frac{d\sqrt{C_l}}{\sqrt{C_l}dt} \sim \frac{2(\sqrt{C_l}(t+\delta t)-\sqrt{C_l}(t))}{(\sqrt{C_l}(t+\delta t)+\sqrt{C_l}(t))}$$

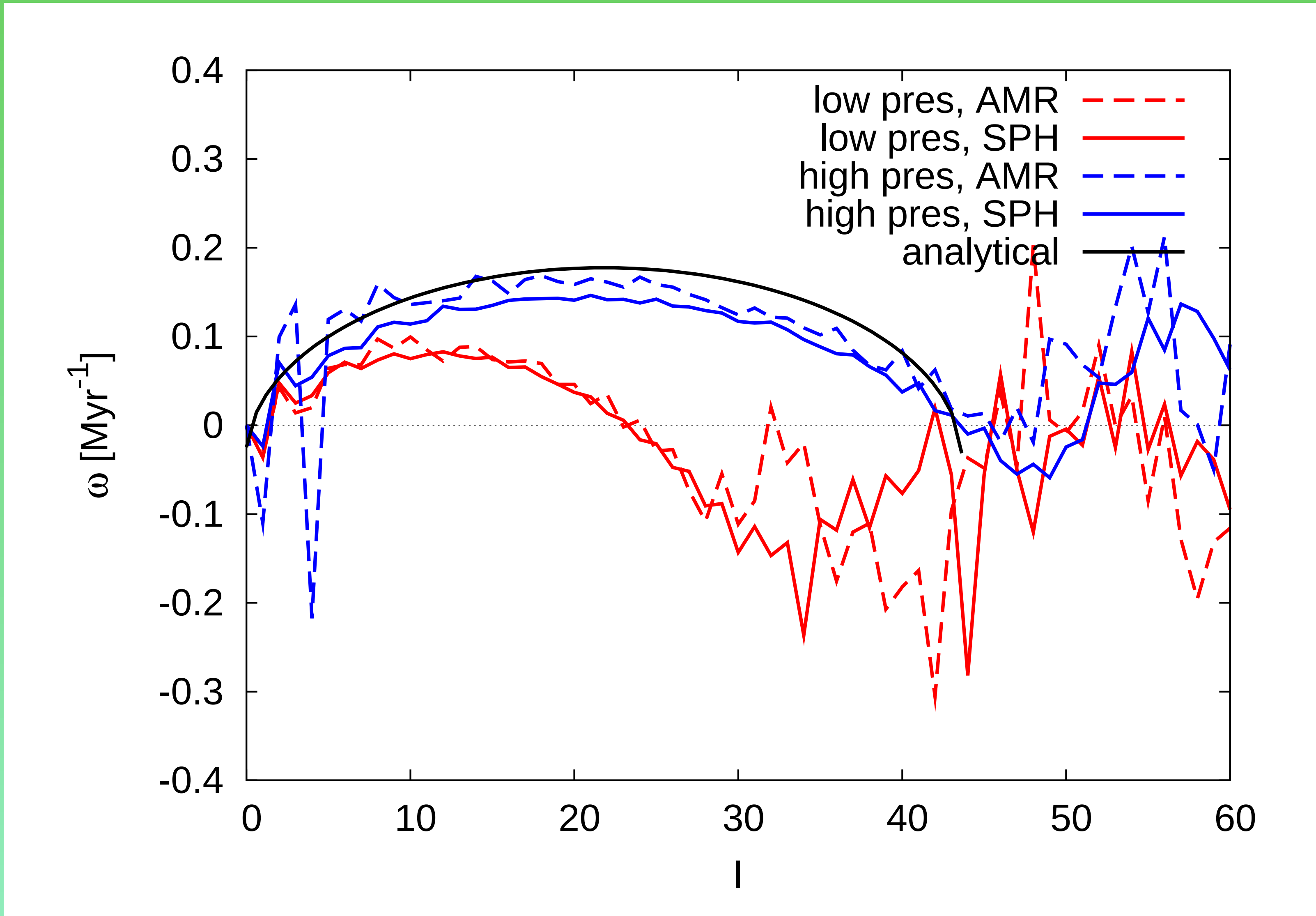
AMR vs. SPH - low pressure



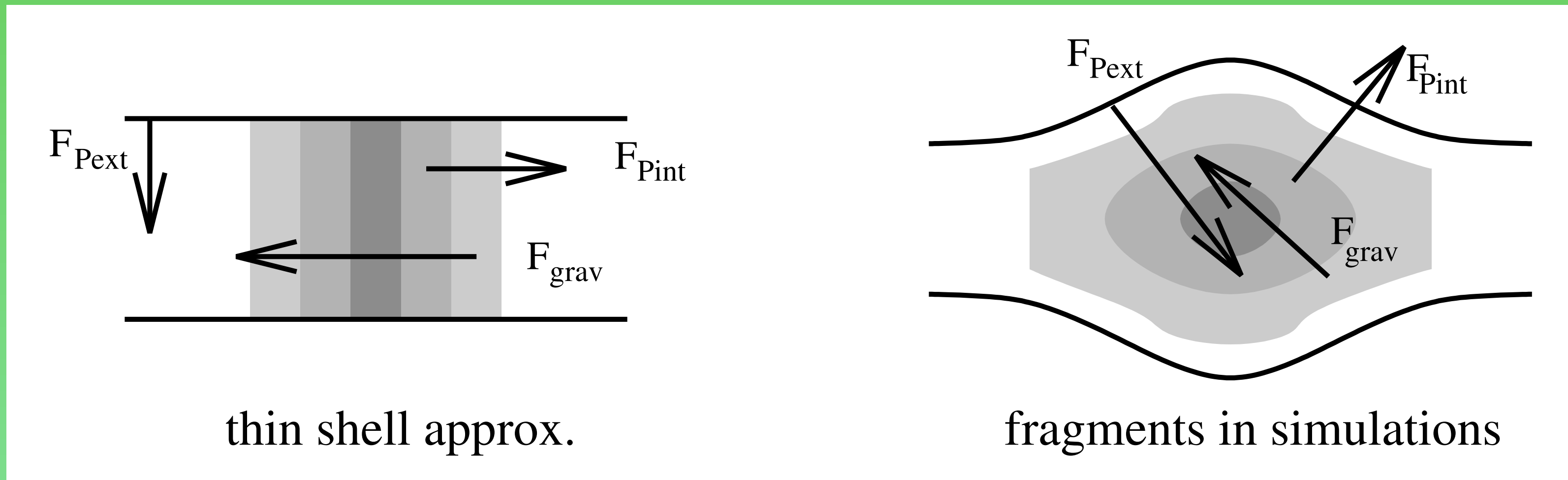
AMR vs. SPH - high pressure

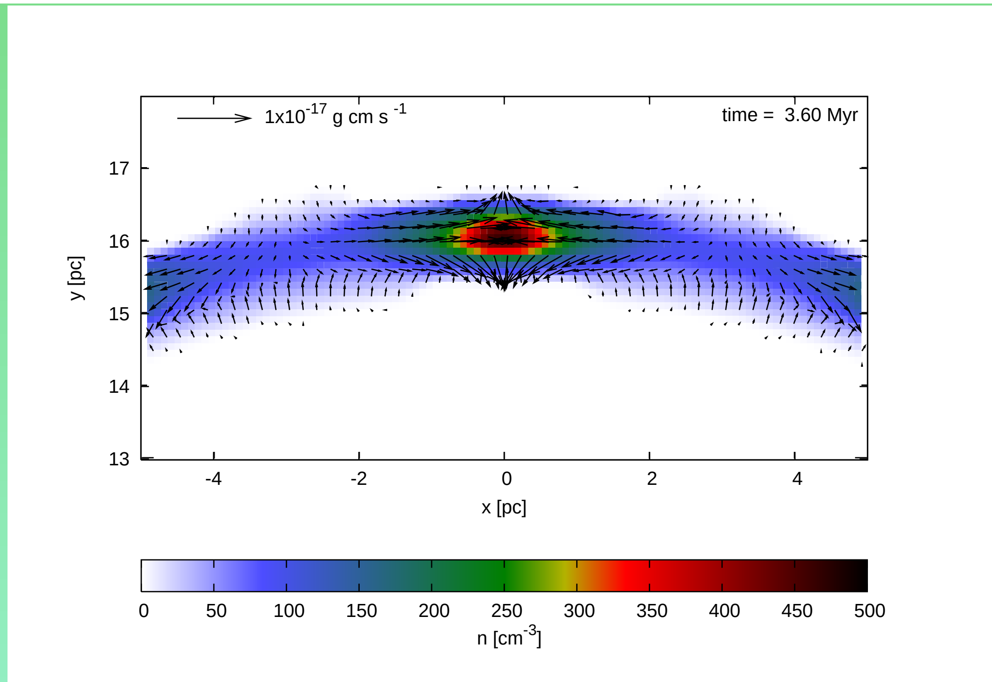


Simulations vs. thin shell



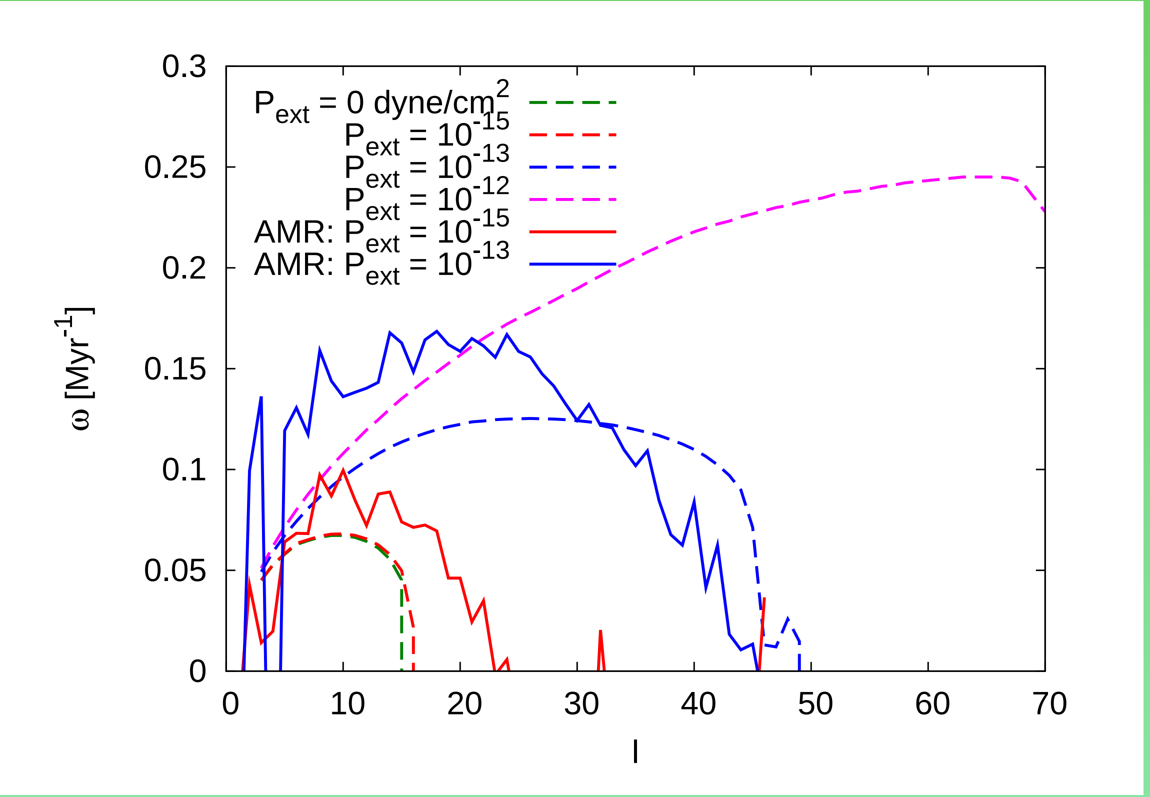
Dependence on the external pressure





Gravitational instability of the thick shell

- fragment modelled as an isolated uniform oblate spheroid (1-zone model) `(Boyd & Whitworth, 2005)`
- EoM solved numerically: fragment collapse time $\rightarrow \omega(l)$



$$\ddot{r} \simeq -\frac{3Gm}{2}\left\{\frac{r\cos^{-1}(z/r)}{(r^2-z^2)^{3/2}} - \frac{(z/r)}{(r^2-z^2)}\right\} - \frac{20\,\pi\,P_{ext}\,r\,z}{3m} + \frac{5\,c_s^2}{r},$$

$$\ddot{z} \simeq -3\,G\,m\,\left\{\frac{1}{(r^2-z^2)} - \frac{z\cos^{-1}(z/r)}{(r^2-z^2)^{3/2}}\right\} - \frac{20\,\pi\,P_{ext}\,r^2}{3m} + \frac{5\,c_s^2}{z}.$$

Summary

- the external pressure is important for the gravitational fragmentation of the expanding shell
- good agreement between AMR and SPH simulations and semi-analytical model, but disagreement with the thin shell approximation
- new tree-based Poisson solver for the FLASH code developed

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Future

- analytical dispersion relation for the thick shell instability
- interaction of GI with hydrodynamic instabilities
- shells driven by stellar winds / ionizing radiation pressure

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