The fragmentation of expanding shells: Thickness matters

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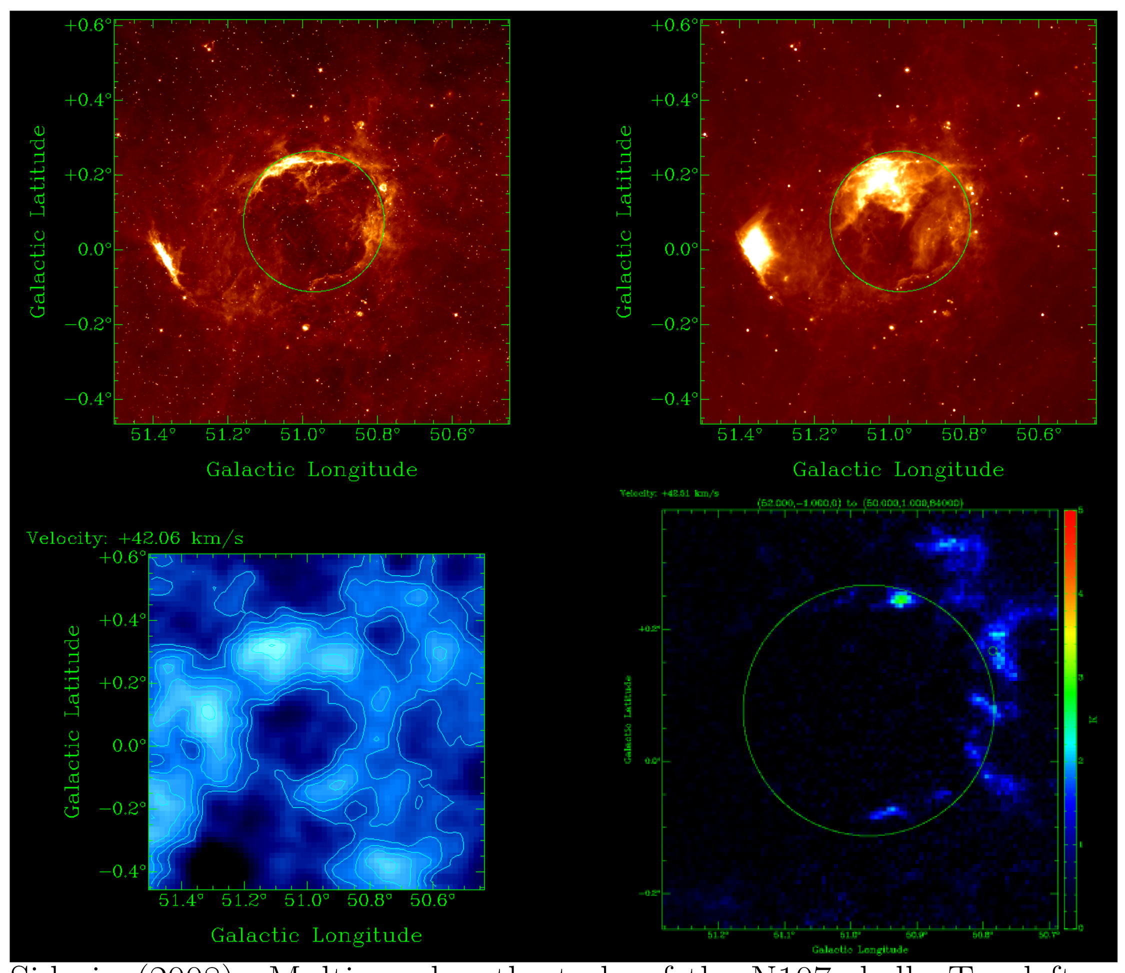
Abstract:

We study analytically the gravitational instability in an expanding shell with a finite thickness. The motivation for this work was a disagreement which we found between 3D hydrodynamic simulations on one hand, and the linear analysis based on the approximation of an infinitesimally thin shell on the other hand. We model the fragment in the shell as a uniform oblate spheroid and derive a new formula for growth rates of surface density perturbations. It takes into account the non-zero shell thickness given by the thermal pressure of the external medium which confines the shell. If the confining pressure is low, only large fragments are unstable. On the other hand, if the confining pressure is high, fragments smaller than predicted by the thin shell analysis become unstable. The new dispersion relation of the thick shell is in a good agreement with AMR and SPH simulations.

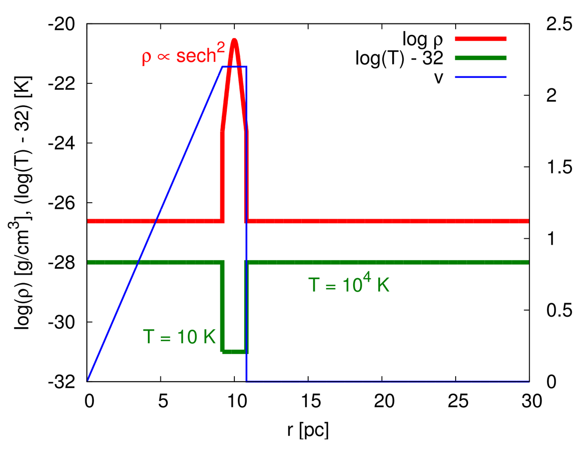


Sidorin (2008). Multiwavelength study of the N107 shell. Top left: 8μm (IRAC), top right: 24μm (MIPS), bottom left: HI (I-GALFA), bottom right: ^{13}CO

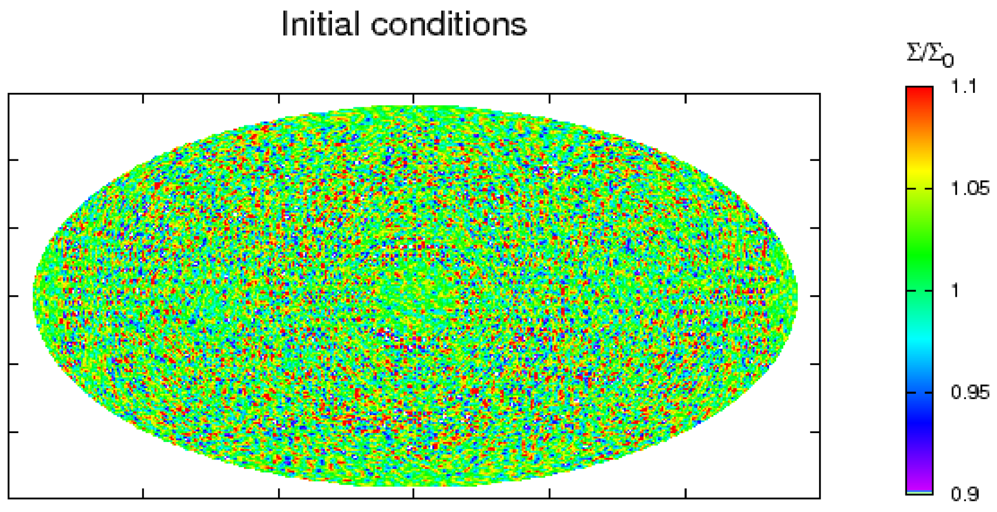
Motivation (see Dale et al. 2009)

Setup of the balistic shell with initial white noise perturbations.

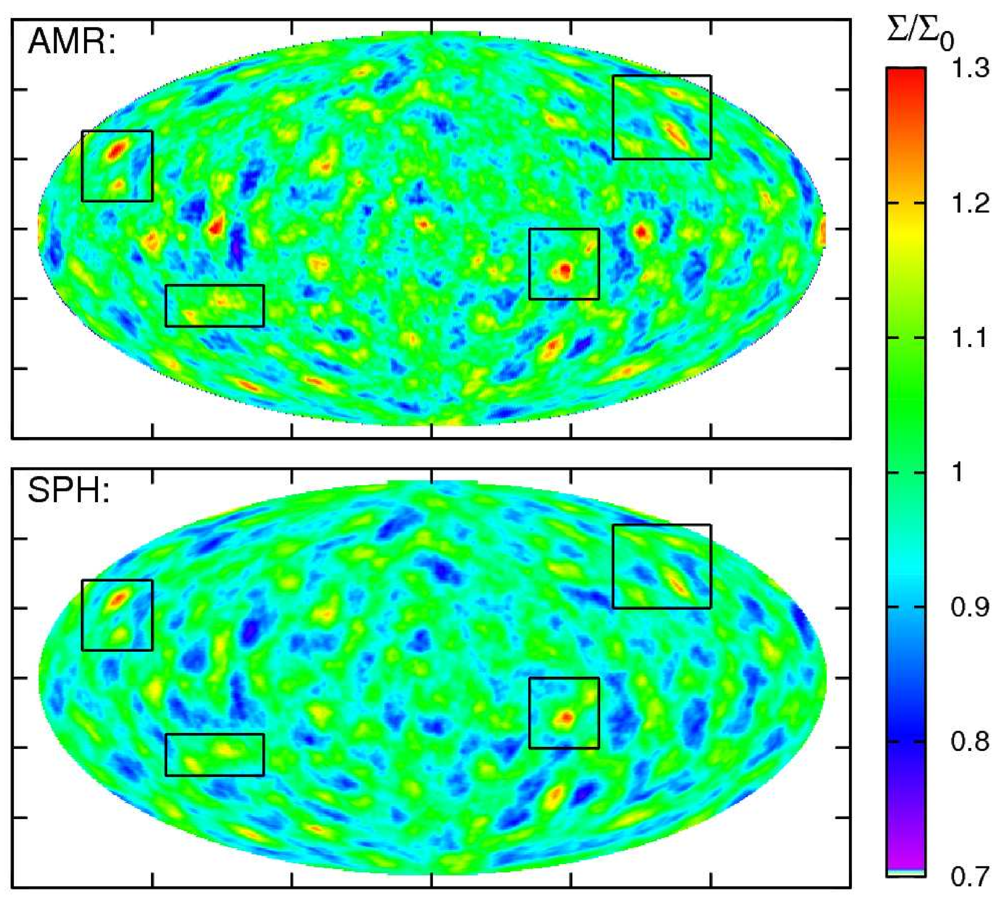
$M_{\rm shell} = 2 \times 10^4$ M$_\odot$
$T_{\rm shell} = 10$ K
$R_{\rm shell,0} = 10$ pc
$V_{\rm shell,0} = 2.2$ km s^{-1}
$R_{\rm shell,max} = 23$ pc
$P_{\rm ext} = 10^{-17}$, 10^{-13}
or 5×10^{-13} dyne cm^{-2}



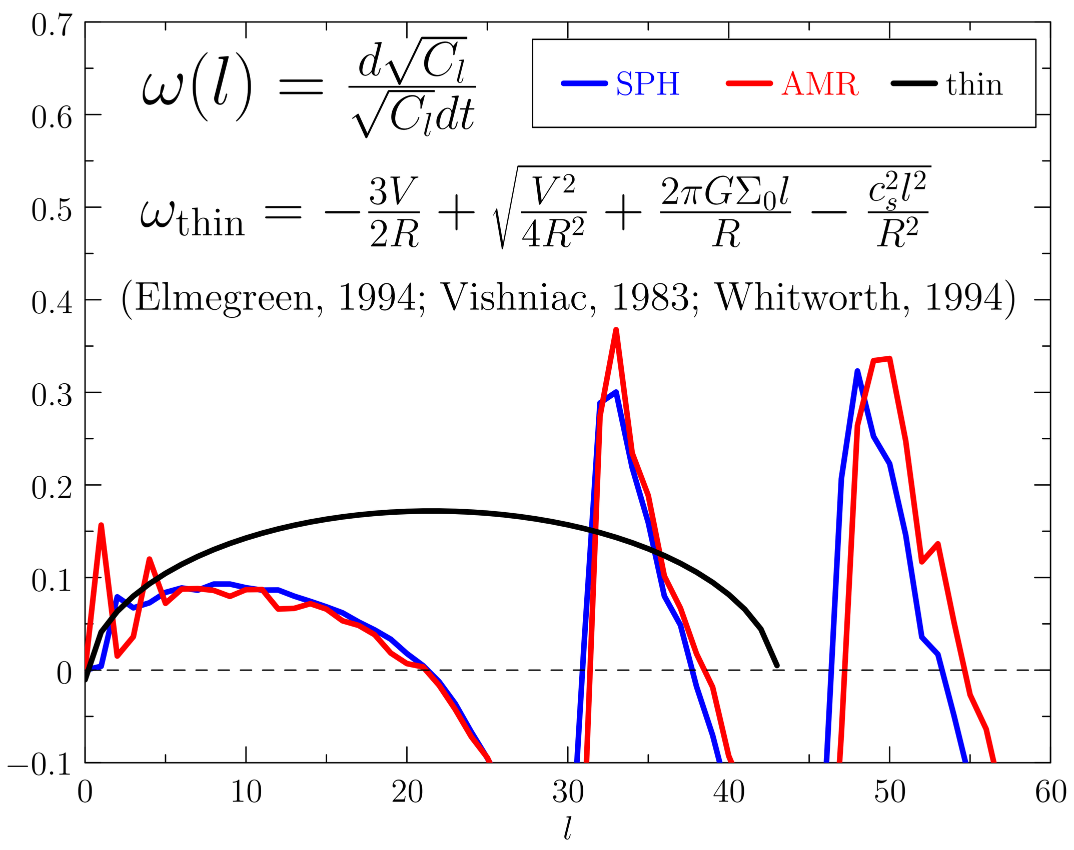
Initial conditions



Surface density in AMR and SPH run agree extremely well with each other.

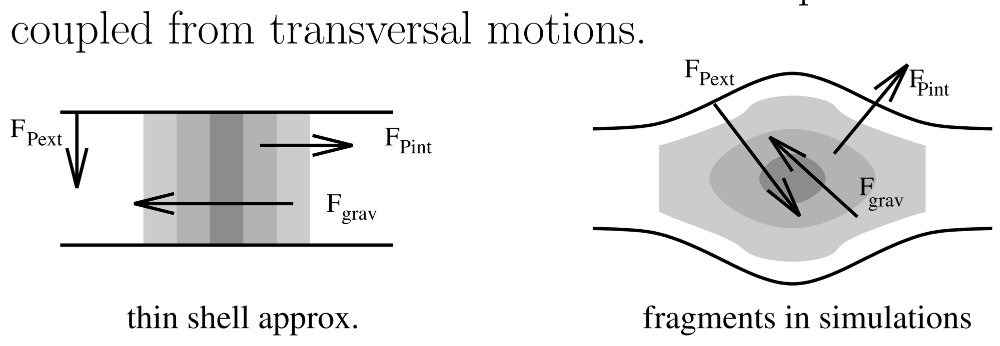


Perturbation growth rate as a function of wavenumber. Both simulations agree again very well, but differ significantly from the thin shell (TS) analysis.

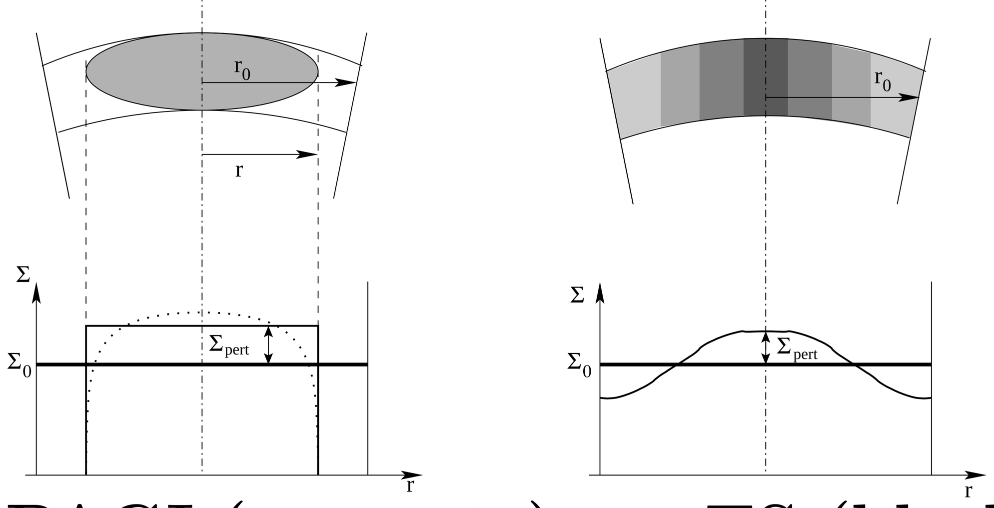


Pressure assisted gravitational instability (see Wünsch et al. 2010)

Thin shell approximation does not work, because it assumes the constant shell thickness and external pressure force decoupled from transversal motions.



New model of the fragment in the thick shell (right) compared to the thin shell approach (left).



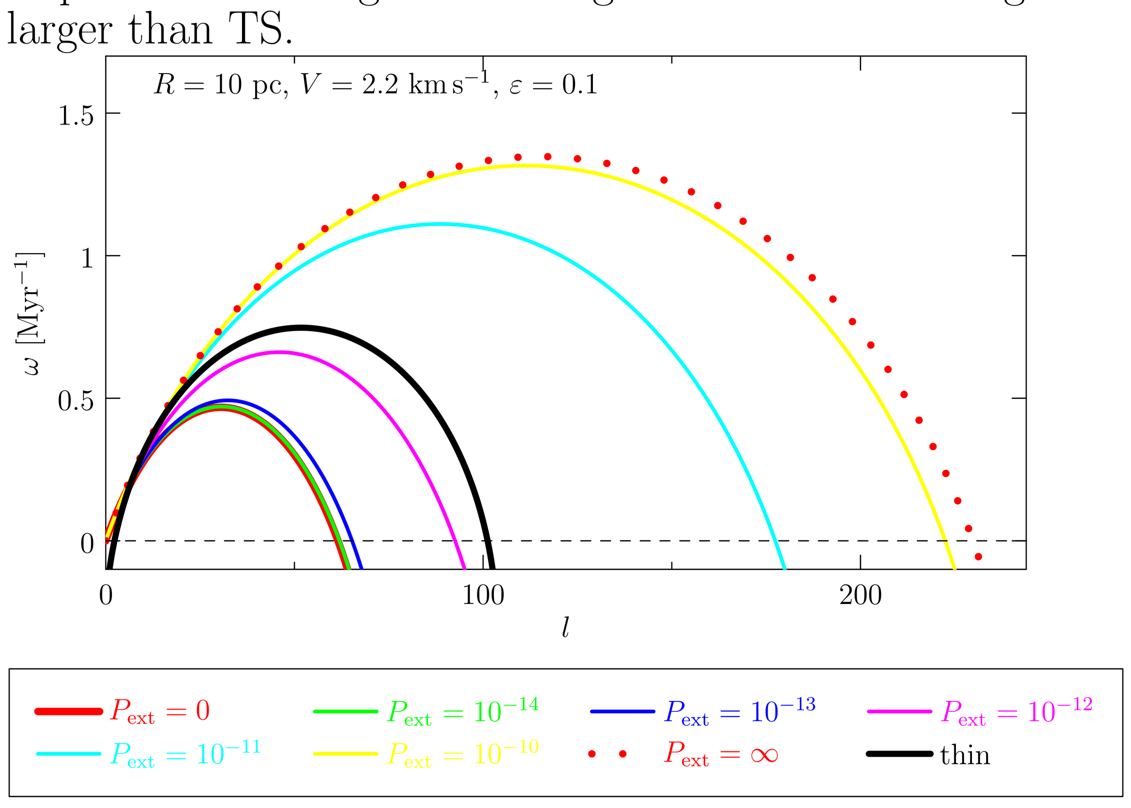
Dispersion relation of the thick shell derived from non-linear equations of motion of the uniform oblate spheroid:

$$\omega_\epsilon = -\frac{V}{2R\epsilon} + \Bigg\{ \underbrace{\frac{V^2}{4R^2\epsilon^2}}_{\rm stretching} + \underbrace{\frac{3G\Sigma_0 l}{4R\epsilon}\left[\frac{\cos^{-1}\beta}{(1-\beta^2)^{3/2}} - \frac{\beta}{1-\beta^2}\right]}_{\rm gravity} + \underbrace{\frac{1-P_{\rm ext}c_s^2 l^2}{3\pi^2R^2\epsilon(2P_{\rm ext}+\pi G\Sigma_0^2)}}_{\rm external\ pres.} - \underbrace{\frac{5c_s^2l^2}{2\pi^2R^2\epsilon}}_{\rm internal\ pres.} \Bigg\}^{1/2}.$$

Compare to thin shell:

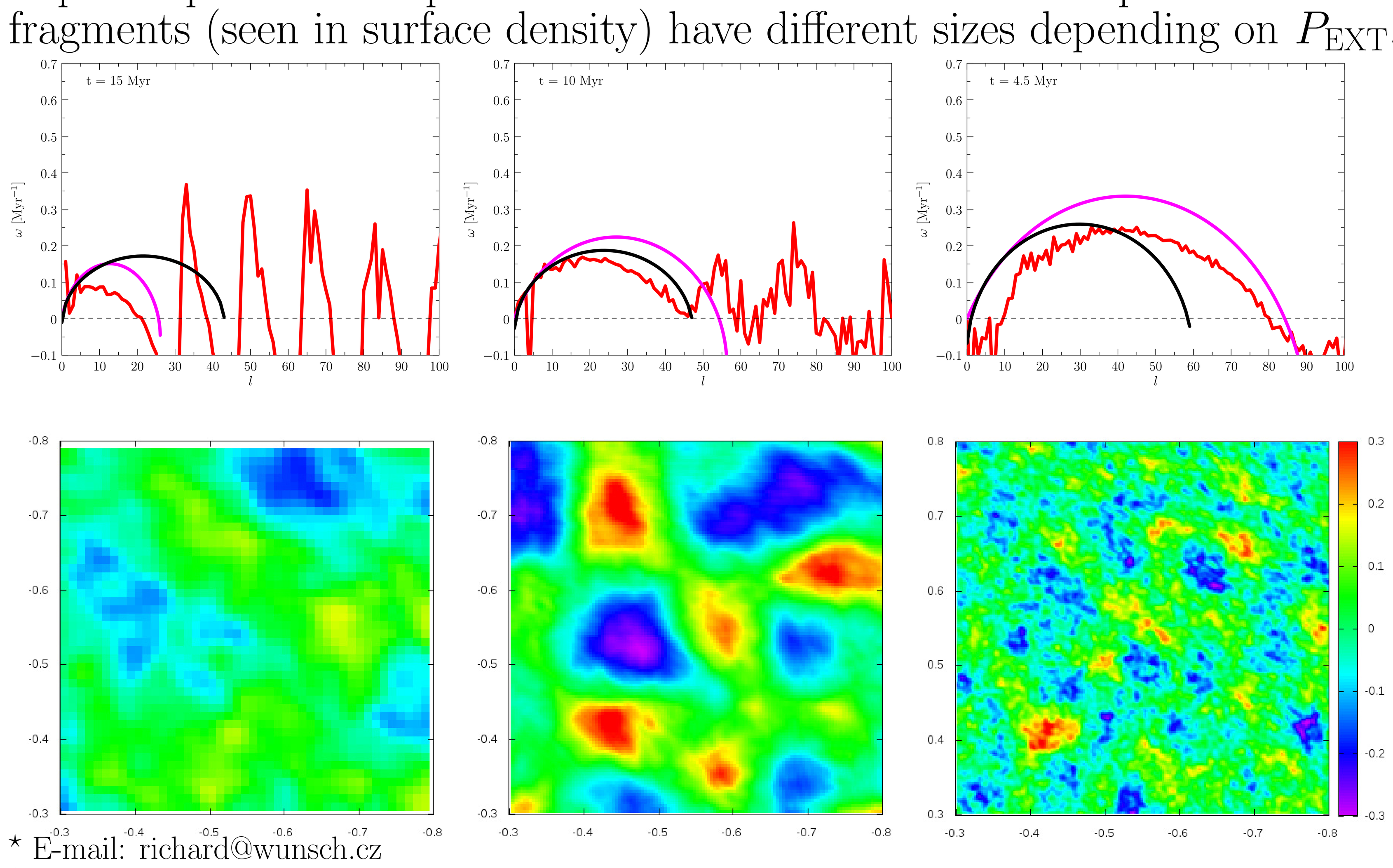
$$\omega_{\rm thin} = -\frac{3V}{2R} + \Bigg(\underbrace{\frac{V^2}{4R^2}}_{\rm stretching} + \underbrace{\frac{2\pi G\Sigma_0 l}{R}}_{\rm gravity} - \underbrace{\frac{c_s^2 l^2}{R^2}}_{\rm internal\ pres.} \Bigg)^{1/2}$$

Dispersion relation of the thick shell depends on the pressure of the external medium $P_{\rm EXT}$. In the limit $P_{\rm EXT} \to \infty$ the PAGI dispersion relation gives the range of unstable wavelength 2.2× larger than TS.



PAGI (magenta) vs. TS (black) vs. Simulations (red)

Top: comparison of dispersion relations for different external pressures. Bottom: fragments (seen in surface density) have different sizes depending on $P_{\rm EXT}$.



Conclusions

- excellent agreement between AMR and SPH, but disagreement with the thin shell approximation
- new instability (PAGI) and dispersion relation (fragment growth rate) for the thick shell embedded in the medium with non-zero pressure
- the PAGI dispersion relation depends on the external pressure; predicts range of unstable wavenumbers different than the one given by the thin shell approximation by factor of 0.6 and 2.2 for $P_{\rm ext} = 0$ and ∞, respectively

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References:
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