



# Thickness matters

## in self-gravitating expanding shells

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### Outline:

#### 1. Observations of shells

→ shells are everywhere

#### 2. Pressure Assisted Gravitational Instability

→ AMR vs. SPH simulations vs. theory

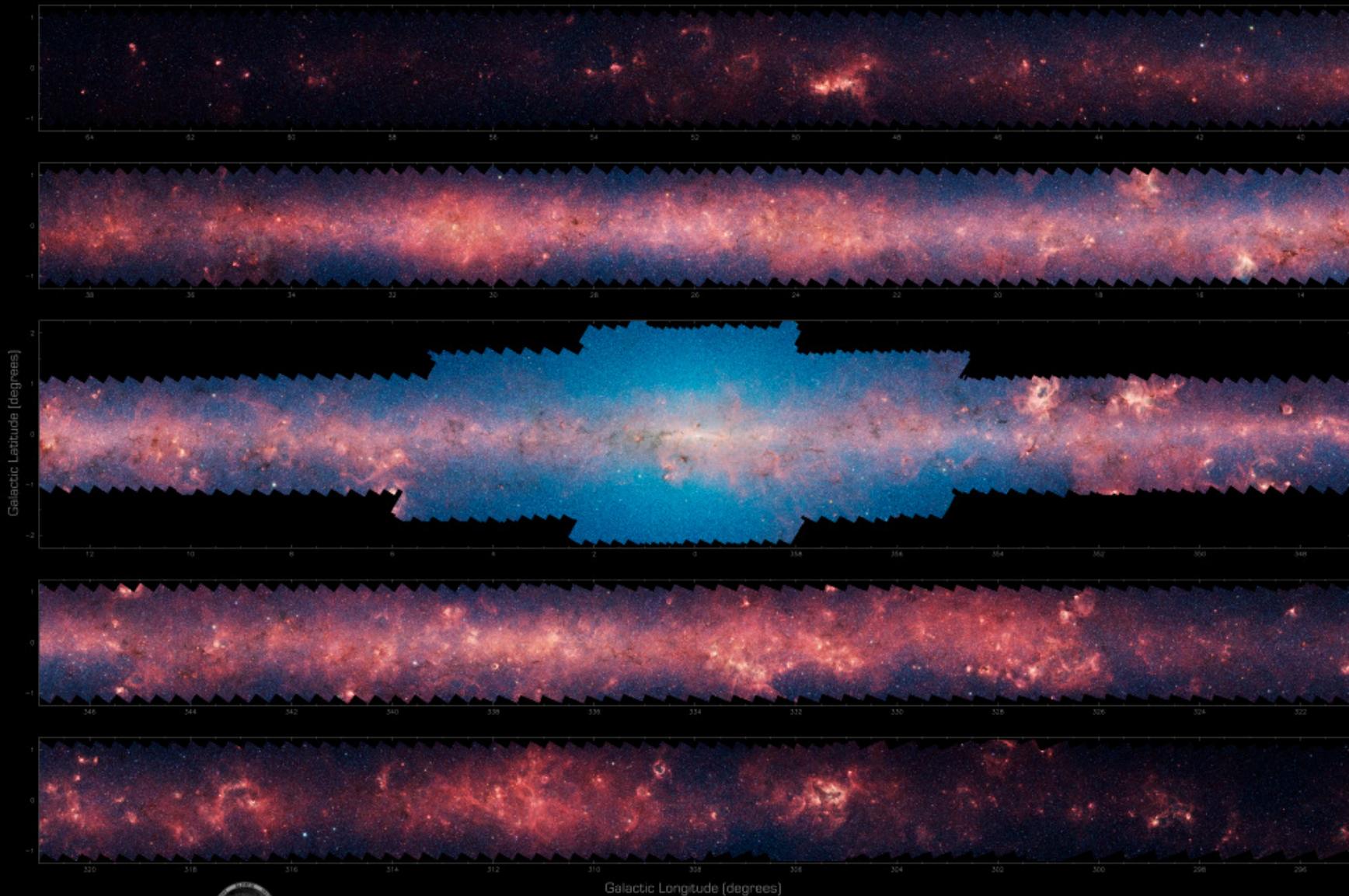
→ Thickness matters

#### 3. Mass spectrum of fragments

→ Oligarchic accretion leads to top-heavy clump mass function

Galactic bubble N107, Credit: Churchwell et al. (2006), Spitzer, GLIMPSE, IRAC,  $8\mu\text{m}$  cont.

# THE INFRARED MILKY WAY: GLIMPSE (3.6–8.0 microns)

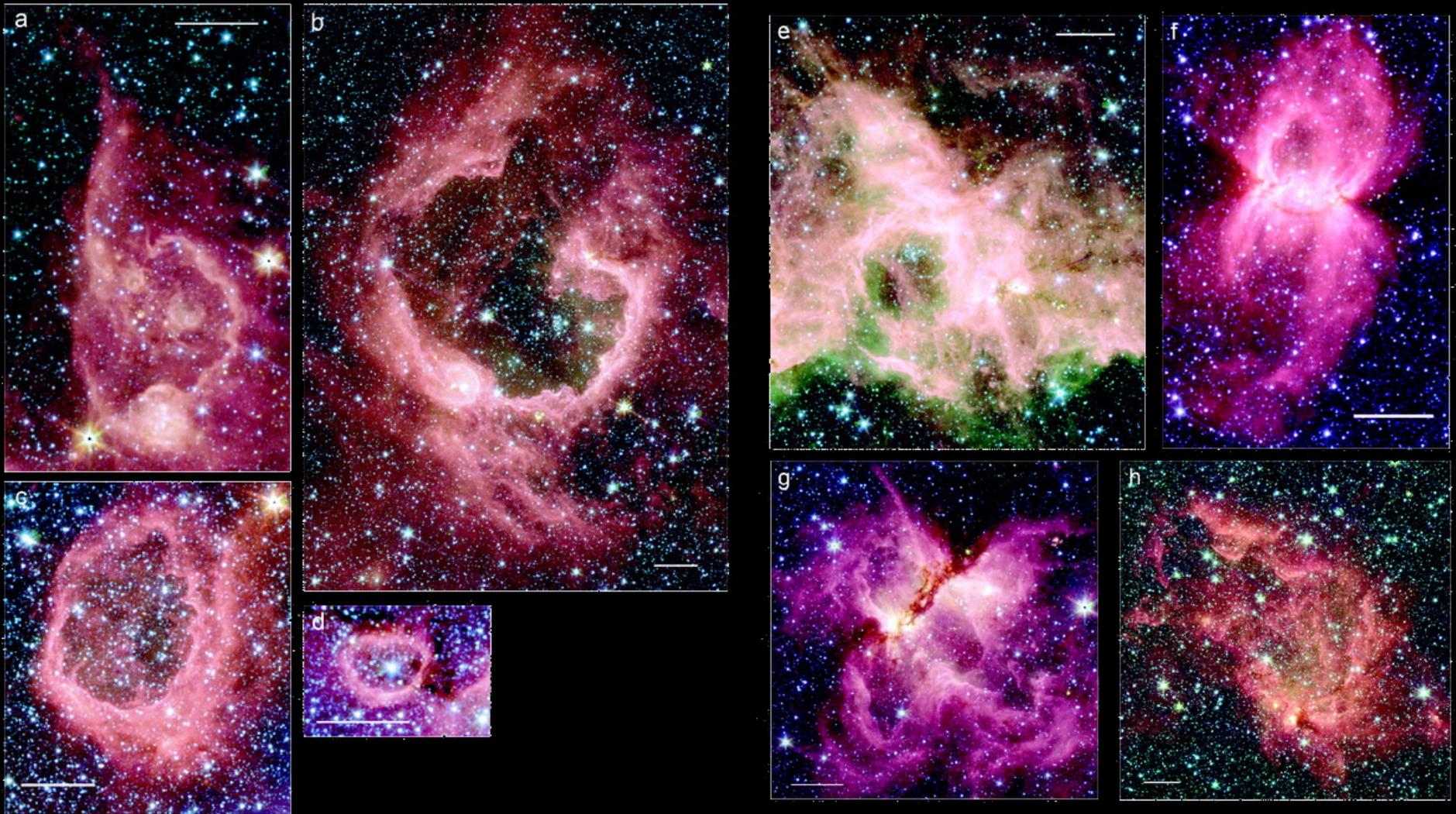


GLIMPSE team: Ed Churchwell (PI), Marilyn Meade, Brian Balser, Pamy Indebetouw, Barbara Whitney, Christine Watson, Bob Benjamin, Steve Brackler, Thomas Robitaille, Stephen Jansen, Doug Wilcots, Mark Wolff, Matt Povich, Tom Berry, Dan Clerman, Martin Cohen, Claudia Cyganowski, Katie Devine, Fabian Heitsch, Jim Jackson, Katharine Johnston, Oleg Kobulecky, John Mathis, Emily Menner, Jeonghee Rho, Marta Sewla, Susan Stolovy, Brian Uppen

Spitzer designed by Thomas Robitaille and Robert Hurt

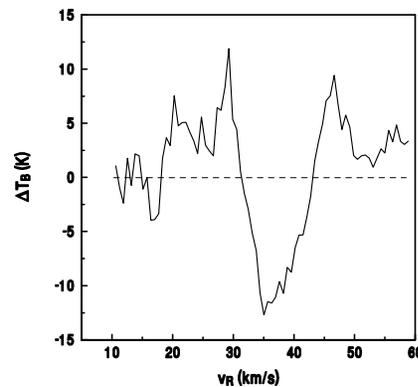
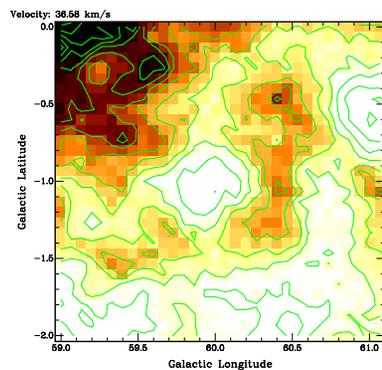
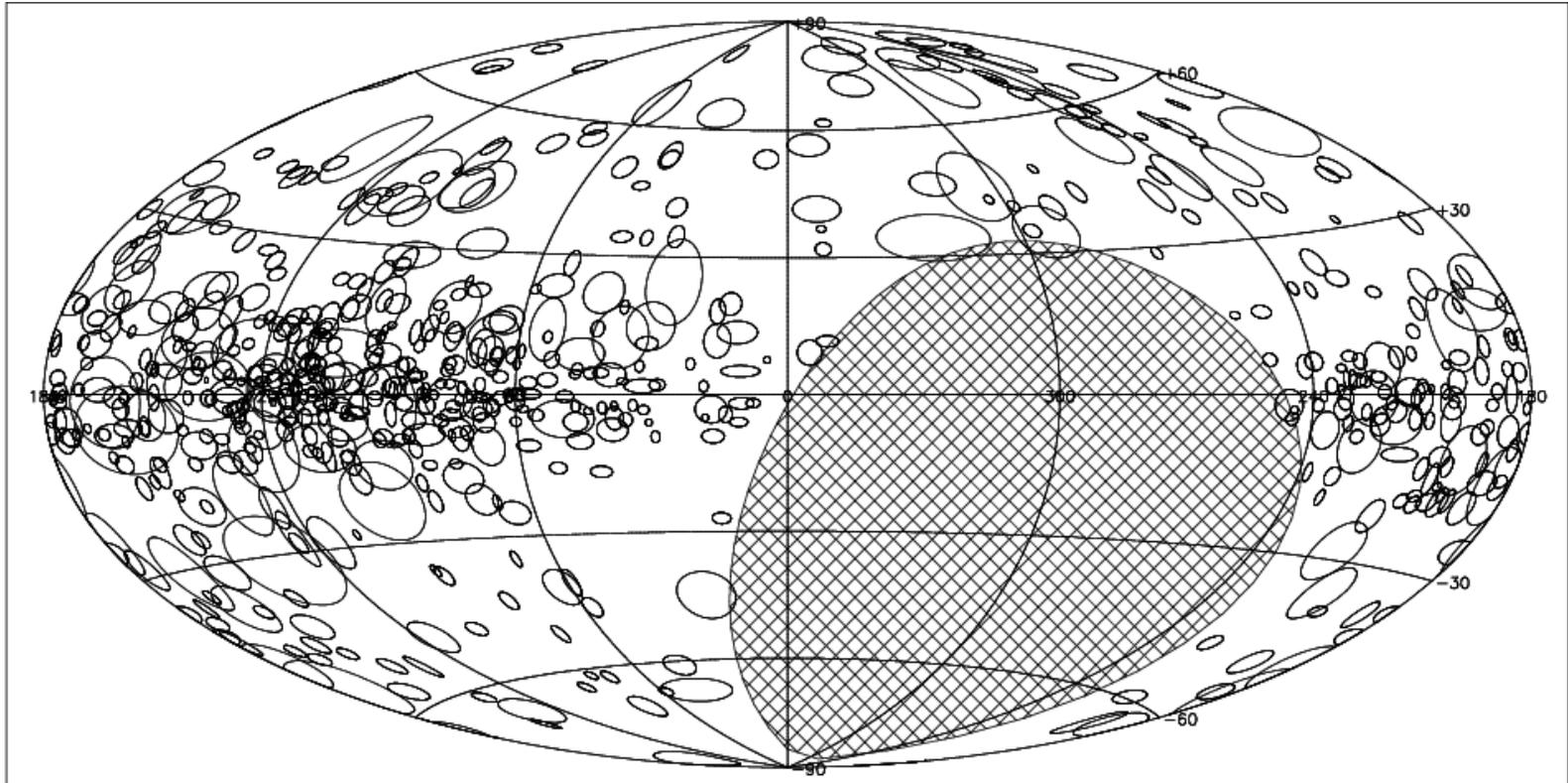
# The bubbling galactic disk

(Churchwell et al., 2006)



Spitzer bands:  $4.5\mu\text{m}$  (blue),  $5.8\mu\text{m}$  (green),  $8\mu\text{m}$  (red)

# Bubbles are everywhere



automatic search for HI shells in  
Leiden-Dwingeloo survey  
→ statistical study of 300 HI  
shells in 2nd quadrant  
(Ehlerová et al. 2005)

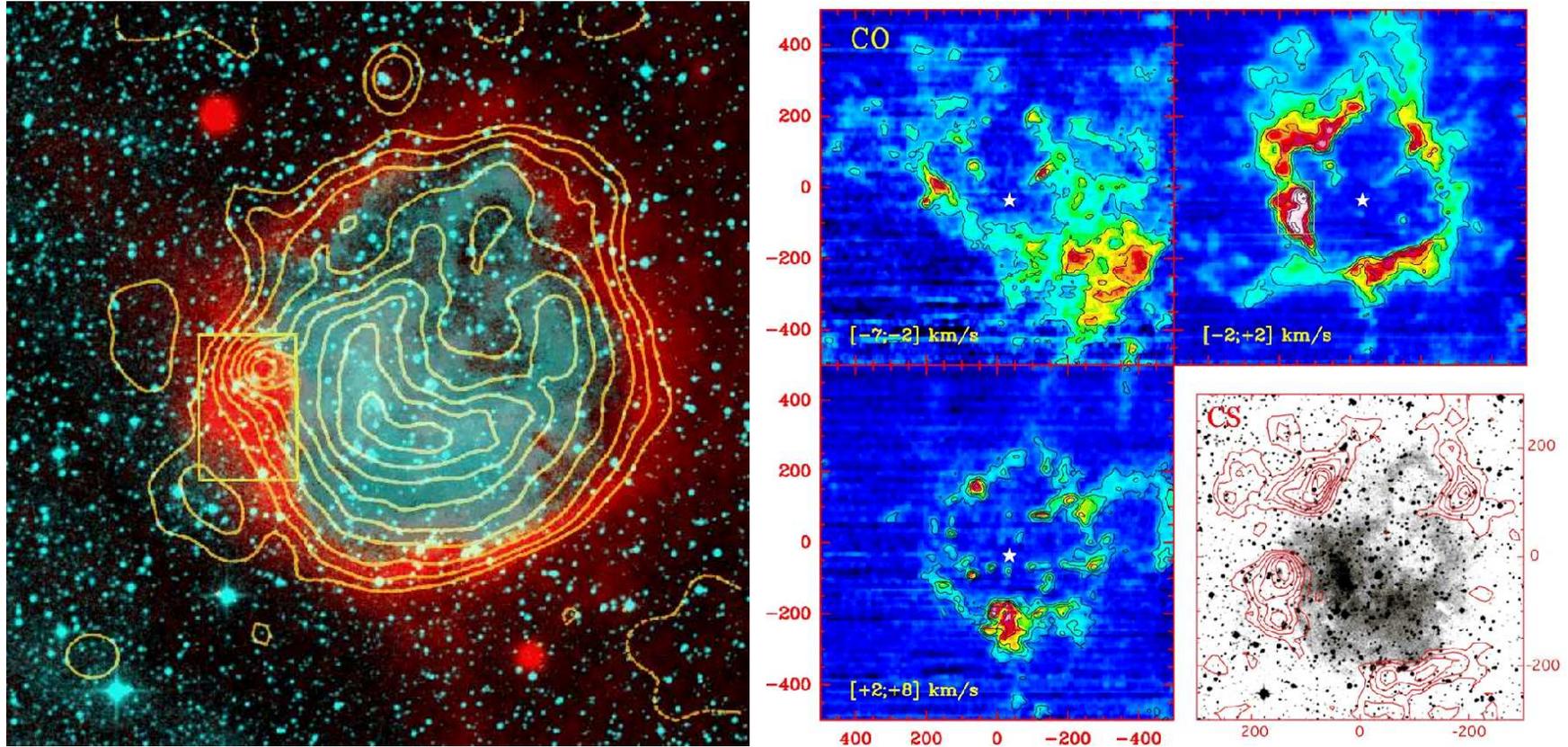
# Bubble properties

- radii:  $r \sim 10 \text{ pc} - 2 \text{ kpc}$
- expansion velocities:  $v_{\text{exp}} \sim 5 - 30 \text{ km/s}$
- formation energy:  $E \sim 10^{51} - 10^{53} \text{ erg}$
- observed in many wavelengths: HI (IGALFA, . . . ), IR (Spitzer/GLIMPSE, . . . ),  $H\alpha$  (WHAM), mm (CO lines), radio cont., X-rays
- observed in MW and many nearby galaxies (LMC, SMC, M31, M33, IC10 . . . )
- origin: OB stars (fossils of expanding HII regions), encounters with HVC or dwarf galaxy, GRB, turbulence and instabilities in ISM
- implications: source of information about ISM, Disk-halo connection, **Triggered star formation (Collect and Collapse)**

# Collect and collapse

(Elmegreen & Lada, 1977)

HII region Sh 104 (Deharveng et al., 2003)

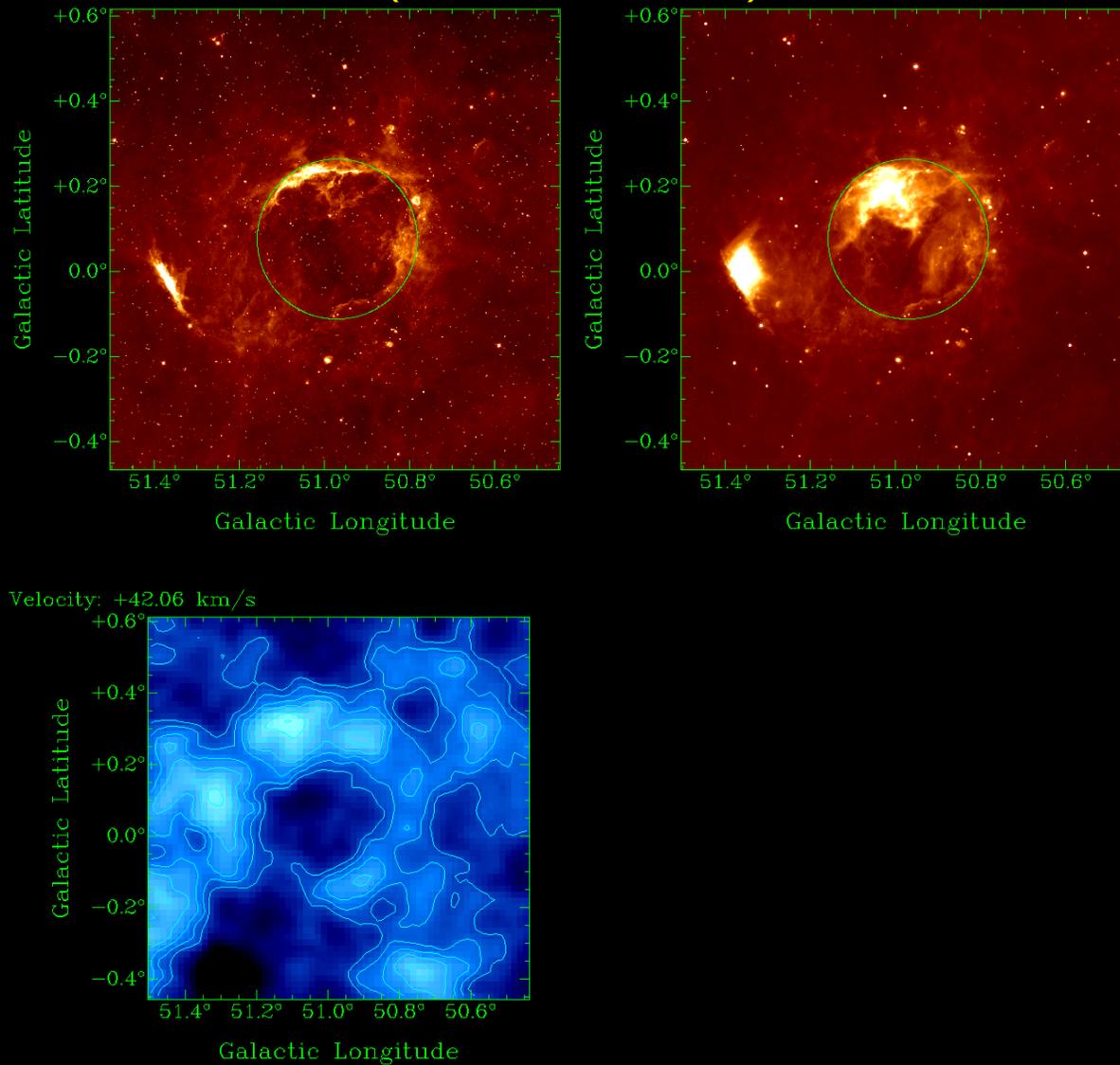


Left: contours (thermal radio continuum 1.46 GHz), red (mid-IR emission - PAHs), turquoise ( $H\alpha$  - ionized gas)

Right: CO molecular line at different velocities

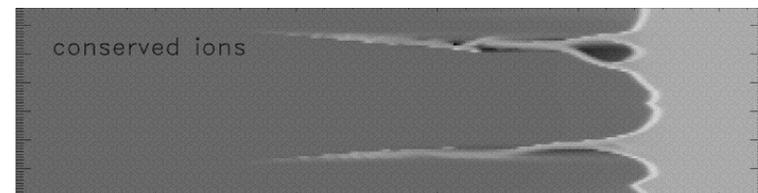
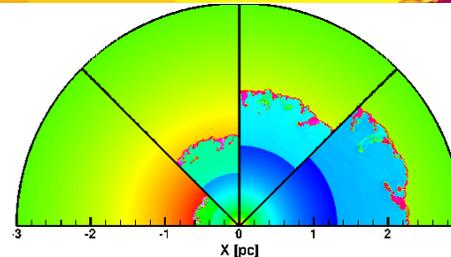
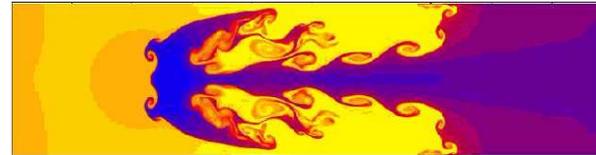
# Multiwavelength study of N107

(Sidorin, 2008)



# Expanding shell instabilities

- gravitational instability (long time-scale)
- Rayleigh-Taylor instability
- Vishniac instability  
(Vishniac, 1983; 1994)
- magnetic field:  
Parker inst. (Parker 1966)  
Wardle inst. (Wardle, 1990)
- ionized shell instability  
(Garcia-Segura & Franco, 1996)



-20.0 -19.5 -19.0 -18.5 -18.0  
log density (g/cm<sup>3</sup>)

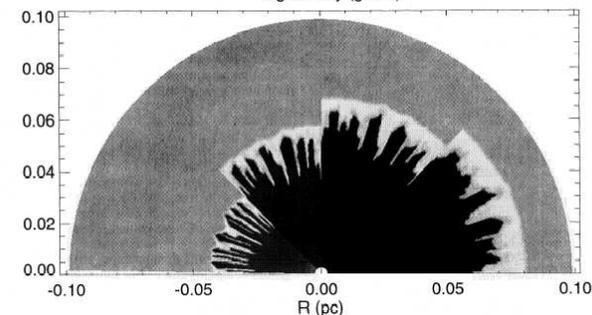
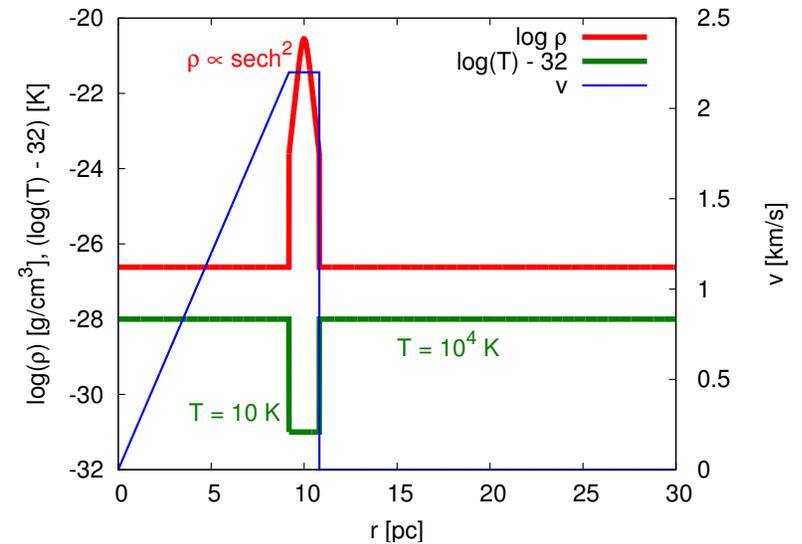


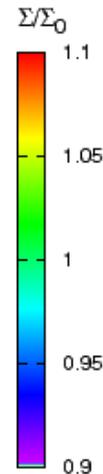
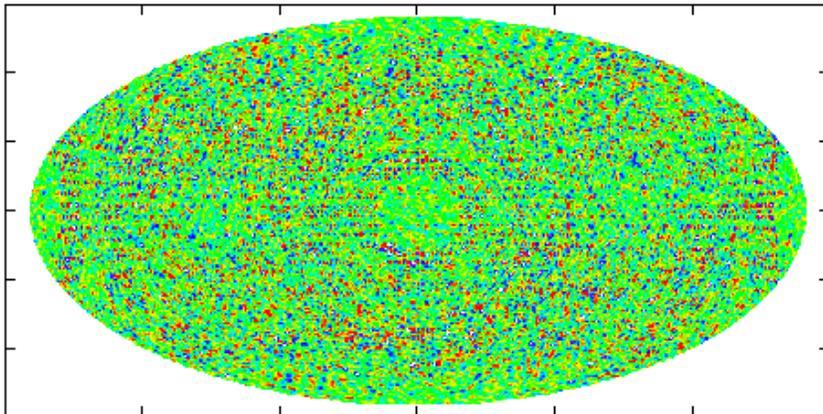
FIG. 6.—Evolution of the I-S front instability for a case with constant density (model UC32). The cooling cutoff is 10<sup>3</sup> K, and the time step is 5 × 10<sup>3</sup> yr. The ambient medium has  $n_0 = 10^3 \text{ cm}^{-3}$  and  $T_0 = 100 \text{ K}$ , and the stellar flux is  $F_* = 10^{48} \text{ s}^{-1}$ . This model is equivalent to the very early stages of model S32, shown in Fig. 8.

# Simulation setup

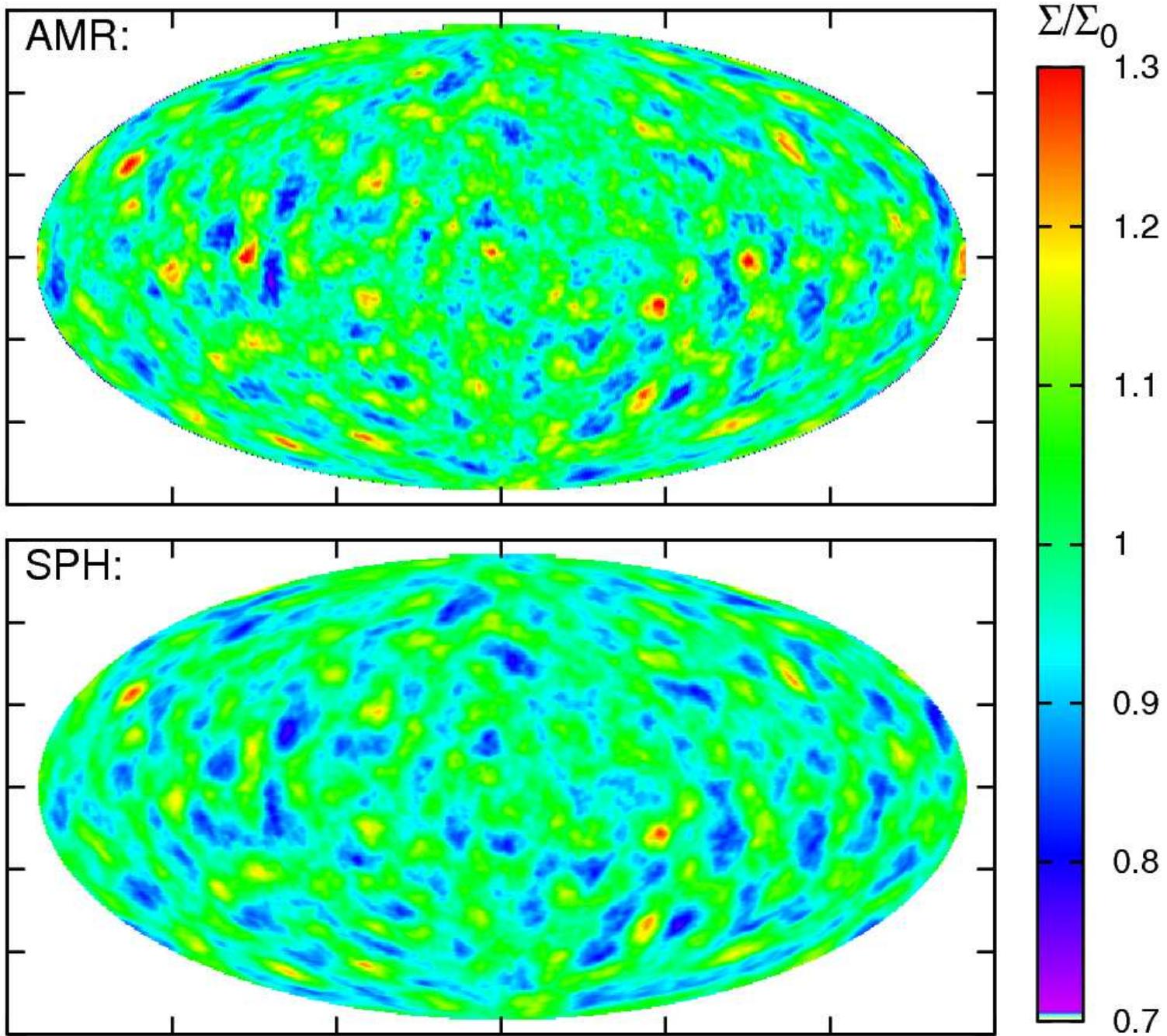
- extremely simplified model to avoid instabilities other than the gravitational one (RT, Vishniac)
- ballistic shell (in a free fall) embedded in a rarefied medium

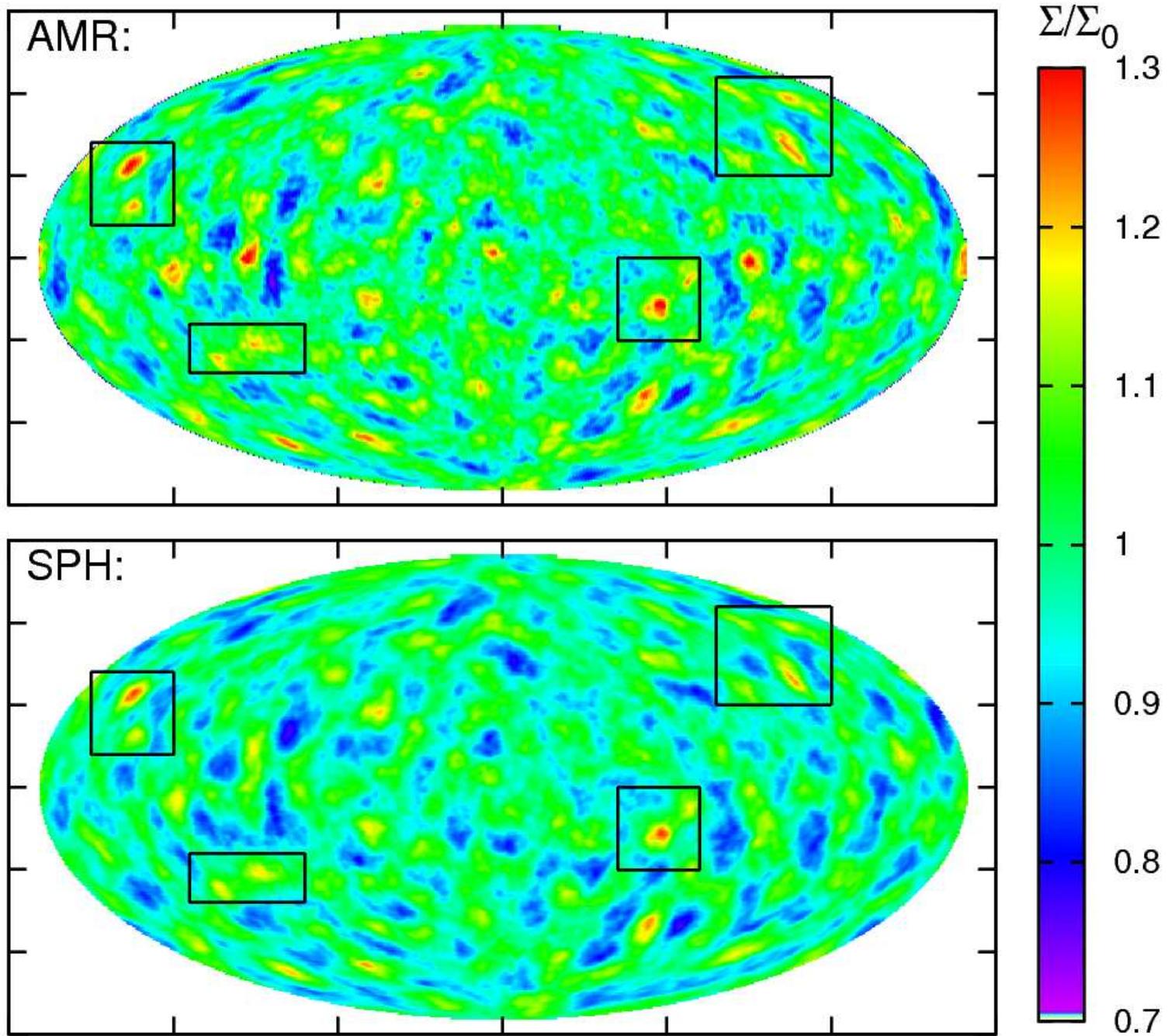


Initial conditions



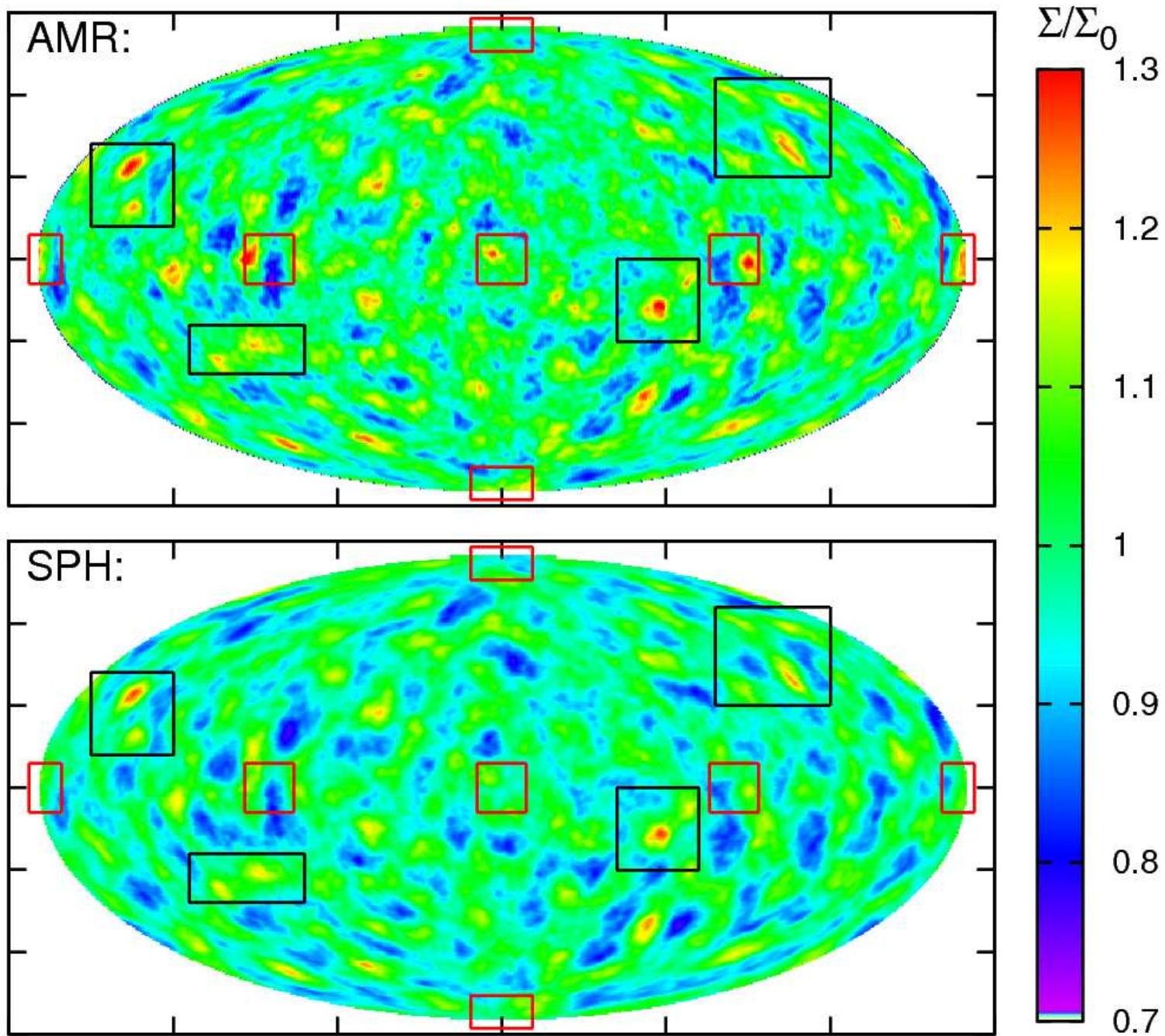
$$\begin{aligned}
 M_{\text{shell}} &= 2 \times 10^4 M_{\odot} \\
 T_{\text{shell}} &= 10 \text{ K} \\
 R_{\text{shell},0} &= 10 \text{ pc} \\
 V_{\text{shell},0} &= 2.2 \text{ km s}^{-1} \\
 R_{\text{shell,max}} &= 23 \text{ pc} \\
 P_{\text{ext}} &= 10^{-17}, 10^{-13} \\
 &\text{or } 5 \times 10^{-13} \text{ dyne cm}^{-2}
 \end{aligned}$$

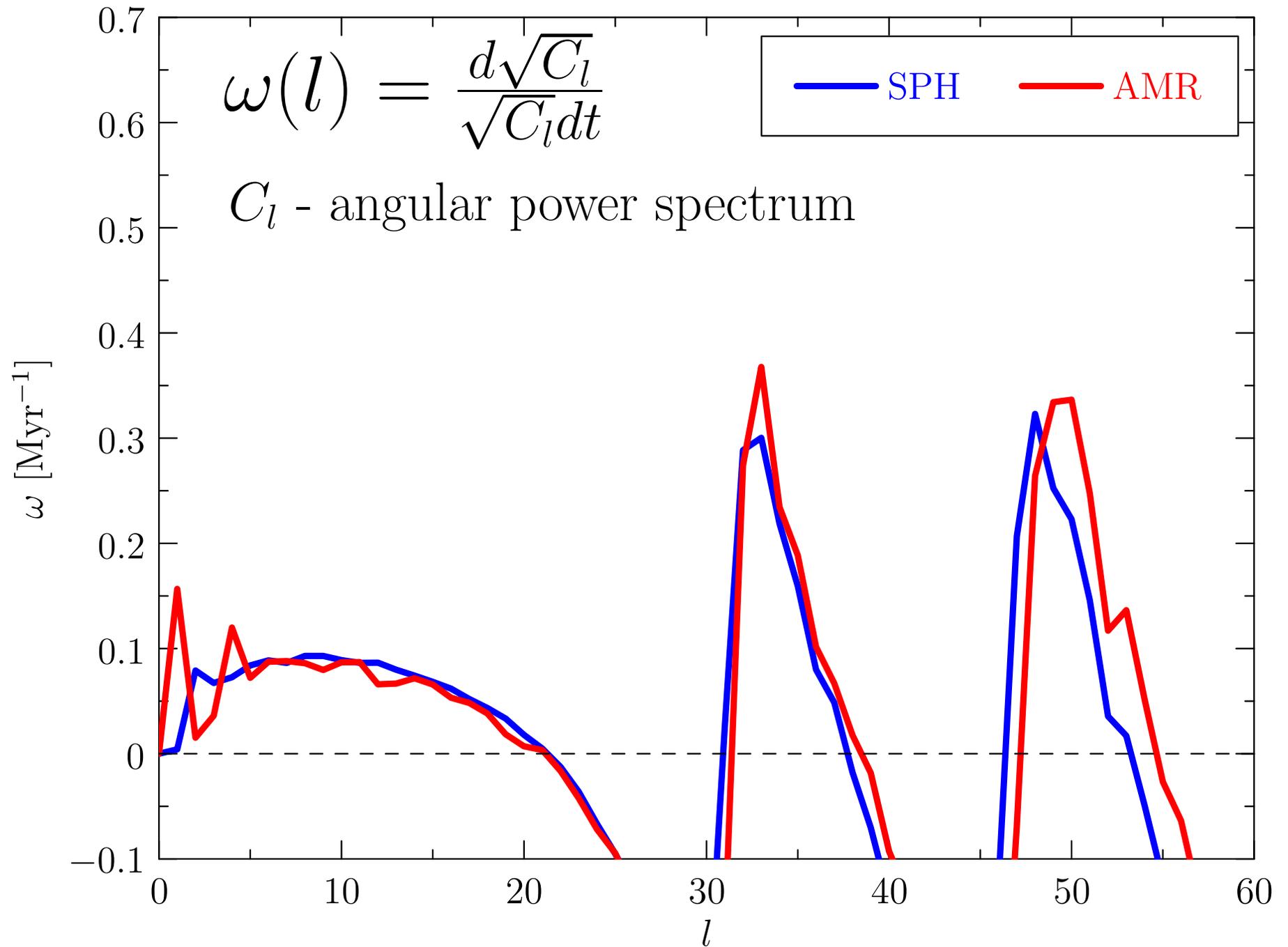


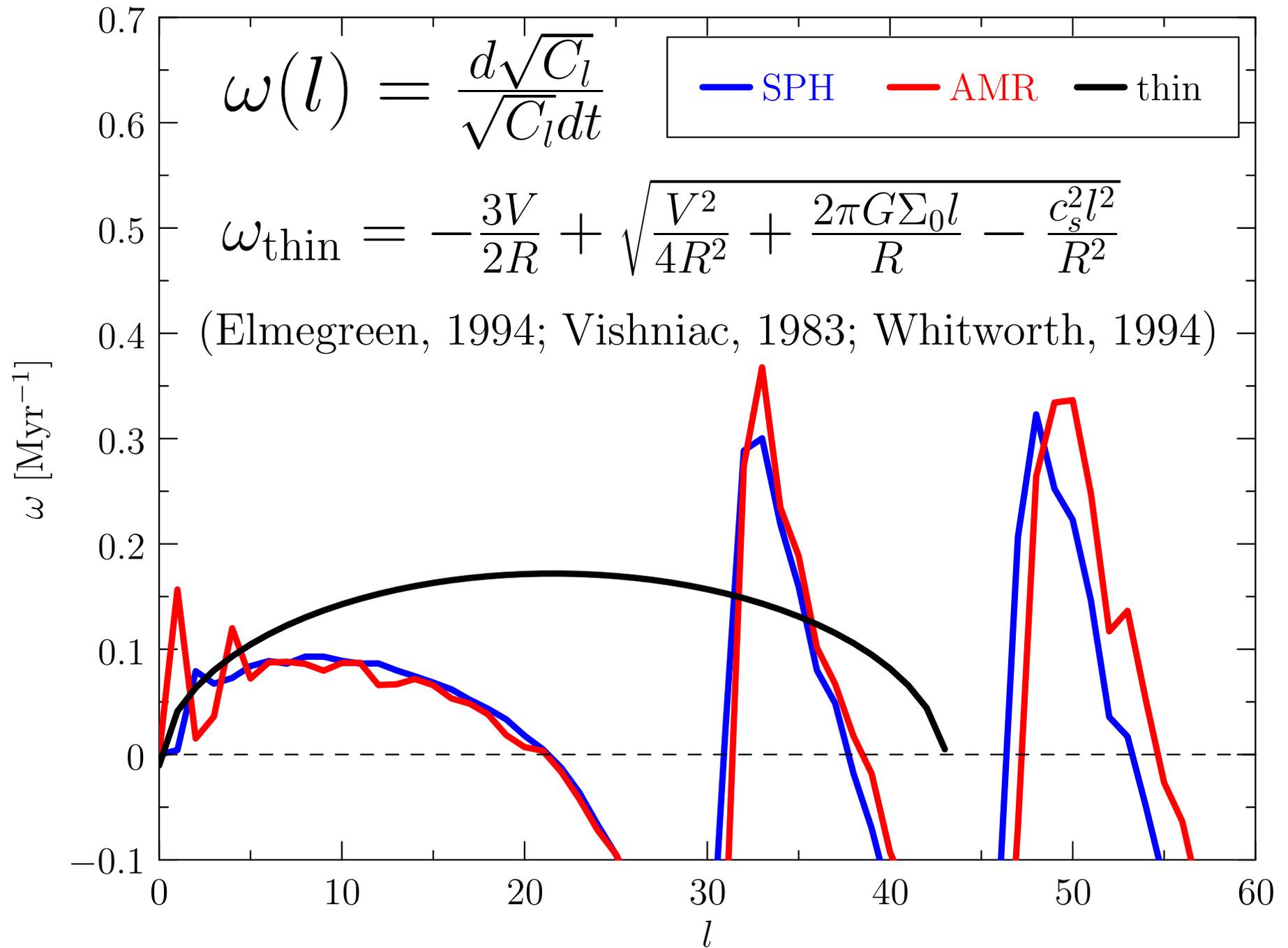


# Find six differences!









# Gravitational instability of thin shell

(Vishniac, 1983; Whitworth et al. 1994; Elmegreen, 1994)

$$\Sigma_0 R \frac{\partial \mathbf{\Omega}}{\partial t} = \overset{\text{pressure}}{-c_s^2 \nabla \Sigma_1} + \overset{\text{gravity}}{\Sigma_0 \nabla \Phi_1} - \overset{\text{stretching}}{\Sigma_0 \mathbf{\Omega} V} - \overset{\text{accretion}}{\cancel{3 \Sigma_0 \mathbf{\Omega} V}}$$

$$\frac{\partial \Sigma_1}{\partial t} = -\Sigma_0 R \nabla_T \cdot \mathbf{\Omega} - 2 \Sigma_1 \frac{V}{R} \leftarrow \text{stretching}$$

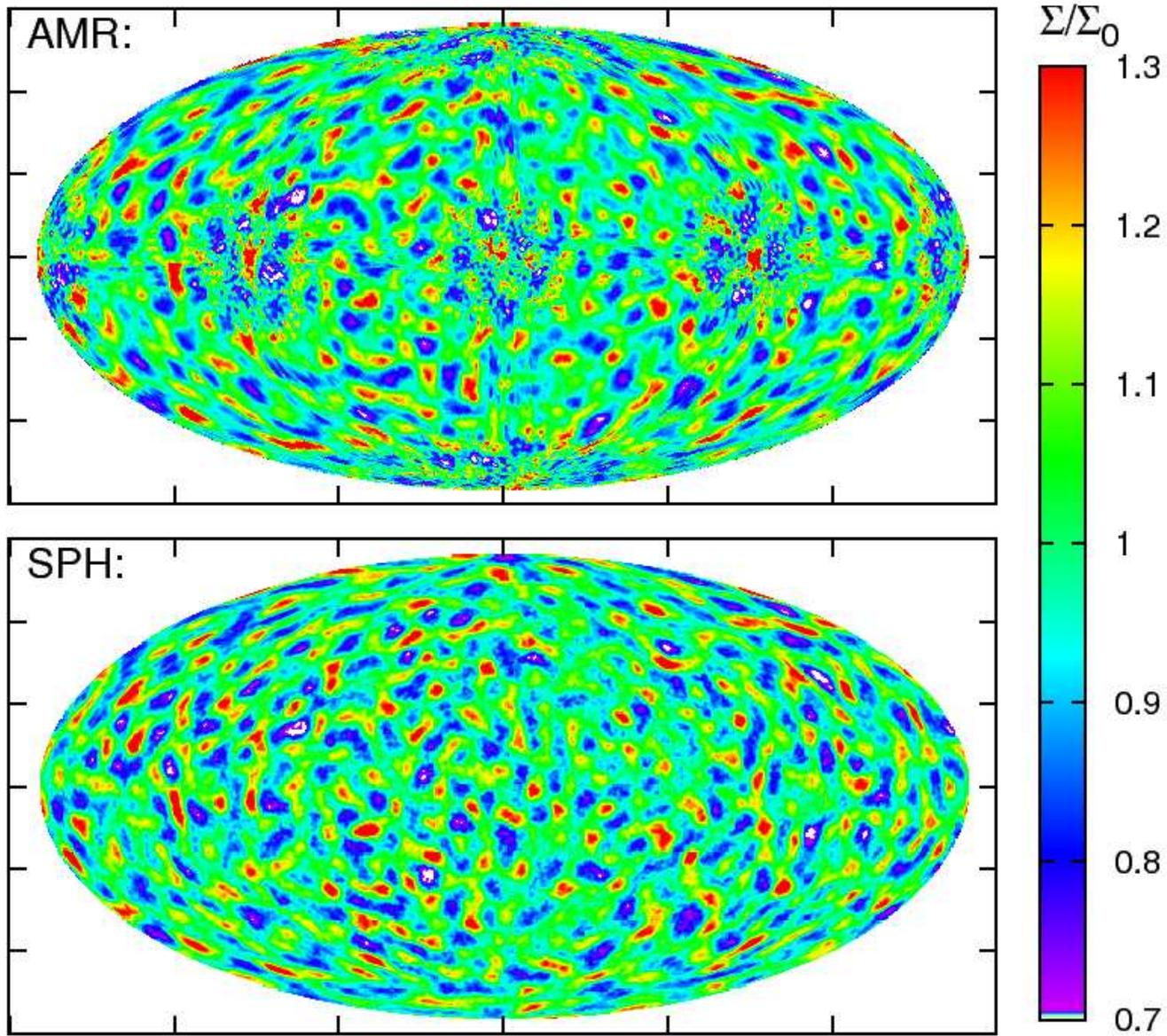
$$\nabla^2 \Phi_1 = 4\pi G \Sigma_1 \delta(r - R)$$

→ linearization, perturbations inserted

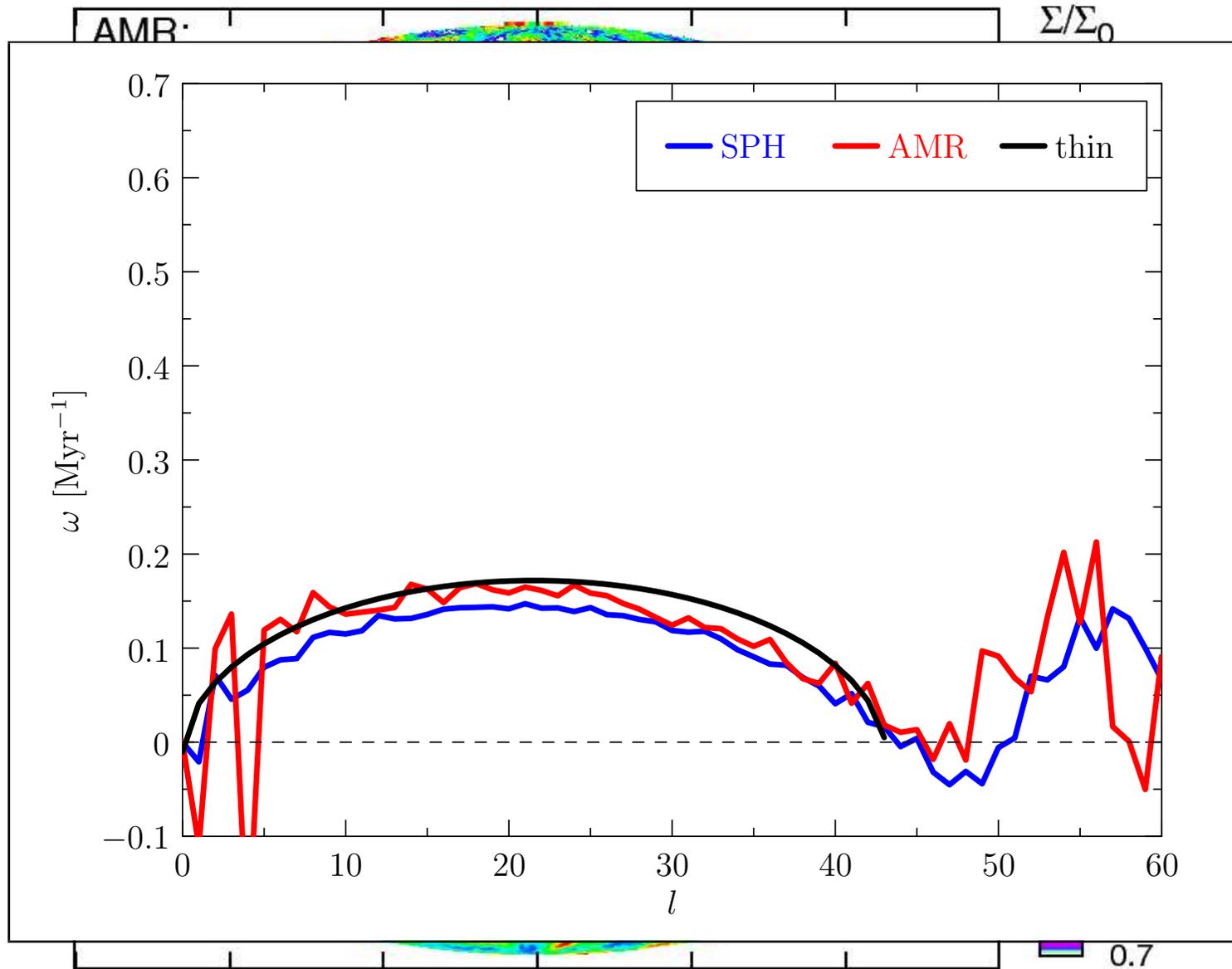
mode with dimensionless wavenumber  $l = kR$  studied:

$$\omega_{\text{thin}}(l) = -\frac{3V}{R} + \sqrt{\frac{V^2}{R^2} + \frac{2\pi G \Sigma_0 l}{R} - \frac{c_s^2 l^2}{R^2}}$$

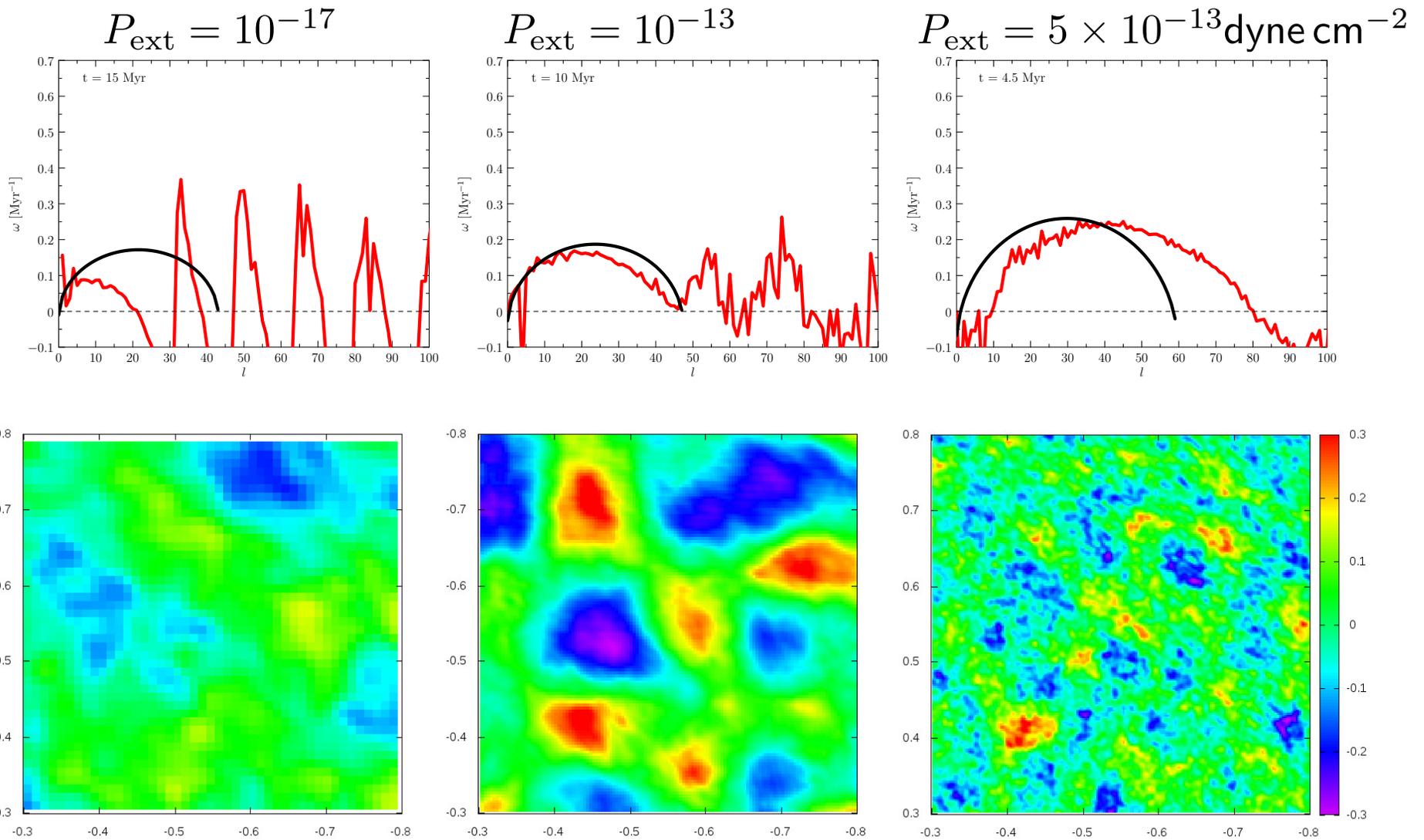
# High pressure ambient medium



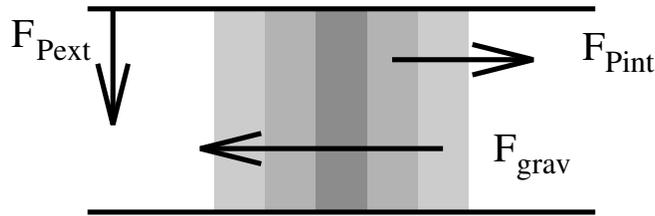
# High pressure ambient medium



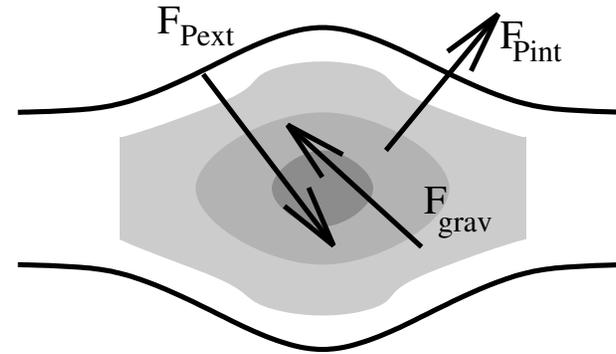
# Dispersion relation depends on $P_{\text{ext}}$ (and hence on the shell thickness)



# Dependence of fragment growth rate on $P_{\text{ext}}$

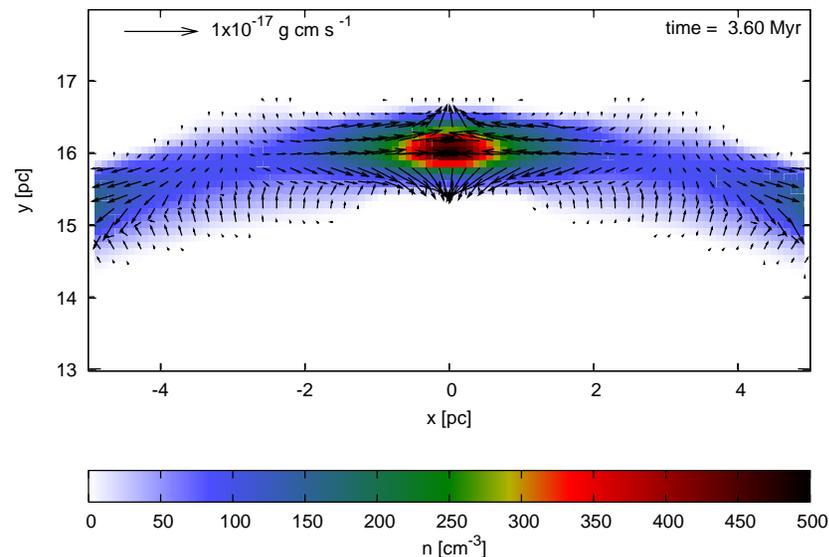


thin shell approx.



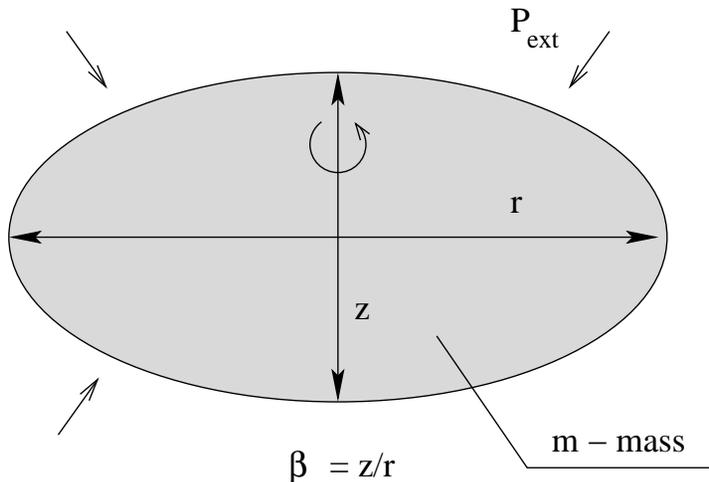
fragments in simulations

- the shell thickness varies across fragments
- external pres. force is not perpendicular to the shell surface



# Uniform Oblate Spheroid

(Boyd & Whitworth, 2005)



Kinetic, gravitational and compressional energy:

$$\mathcal{K} = \frac{m}{10} (2\dot{r}^2 + \dot{z}^2)$$

$$\mathcal{G} = -\frac{3Gm^2}{5} \frac{\cos^{-1}(z/r)}{(r^2 - z^2)^{1/2}}$$

$$\frac{d\mathcal{B}}{dV} = P_{\text{ext}} - P_{\text{int}} = P_{\text{ext}} - \frac{m c_s^2}{V}$$

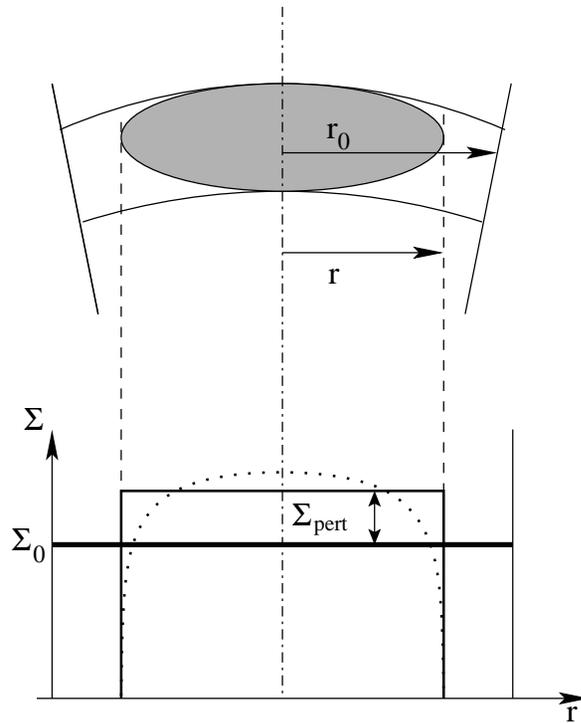
- energy conservation:  $\mathcal{E} = \mathcal{K} + \mathcal{G} + \mathcal{B}$
- differentiate with resp. to time:

$$\left\{ \frac{2m\ddot{r}}{5} + \frac{d\mathcal{G}}{dr} + \frac{d\mathcal{B}}{dr} \right\} \dot{r} + \left\{ \frac{m\ddot{z}}{5} + \frac{d\mathcal{G}}{dz} + \frac{d\mathcal{B}}{dz} \right\} \dot{z} = 0.$$

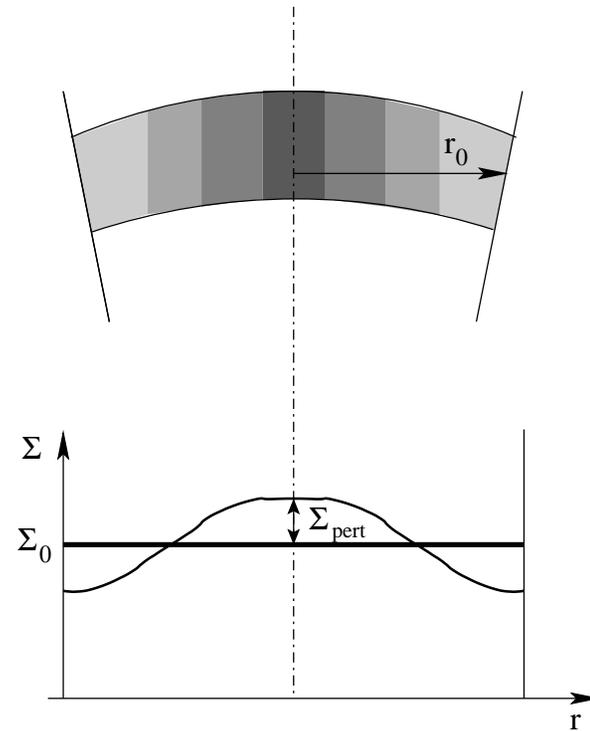
- $\dot{r}$  and  $\dot{z}$  independent:

$$\ddot{r} = -\frac{3Gm}{2r^2} \left[ \frac{\cos^{-1} \beta}{(1 - \beta^2)^{3/2}} - \frac{\beta}{1 - \beta^2} \right] - \frac{20\pi P_{\text{ext}} r z}{3m} + \frac{5c_s^2}{r}$$

# UOS vs. thin shell



- $z$  given by hydrostatic equilibrium
- non-linear eqs. of motion
- homogeneous ellipsoid



- no vertical structure (inf. thin shell)
- linearized hydrodyn. eqs.
- sinusoidal perturbations

# Dispersion relation of the thick shell

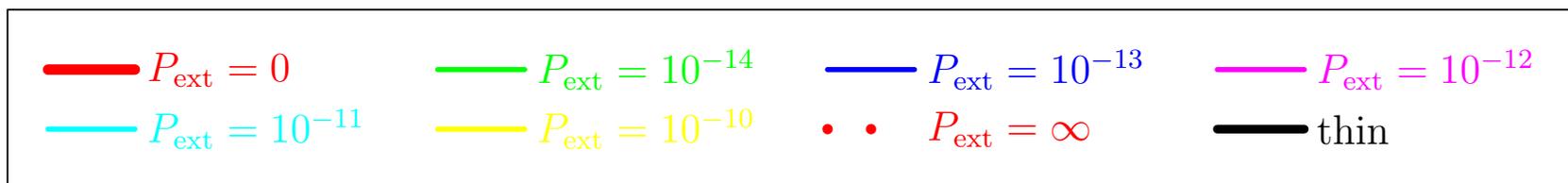
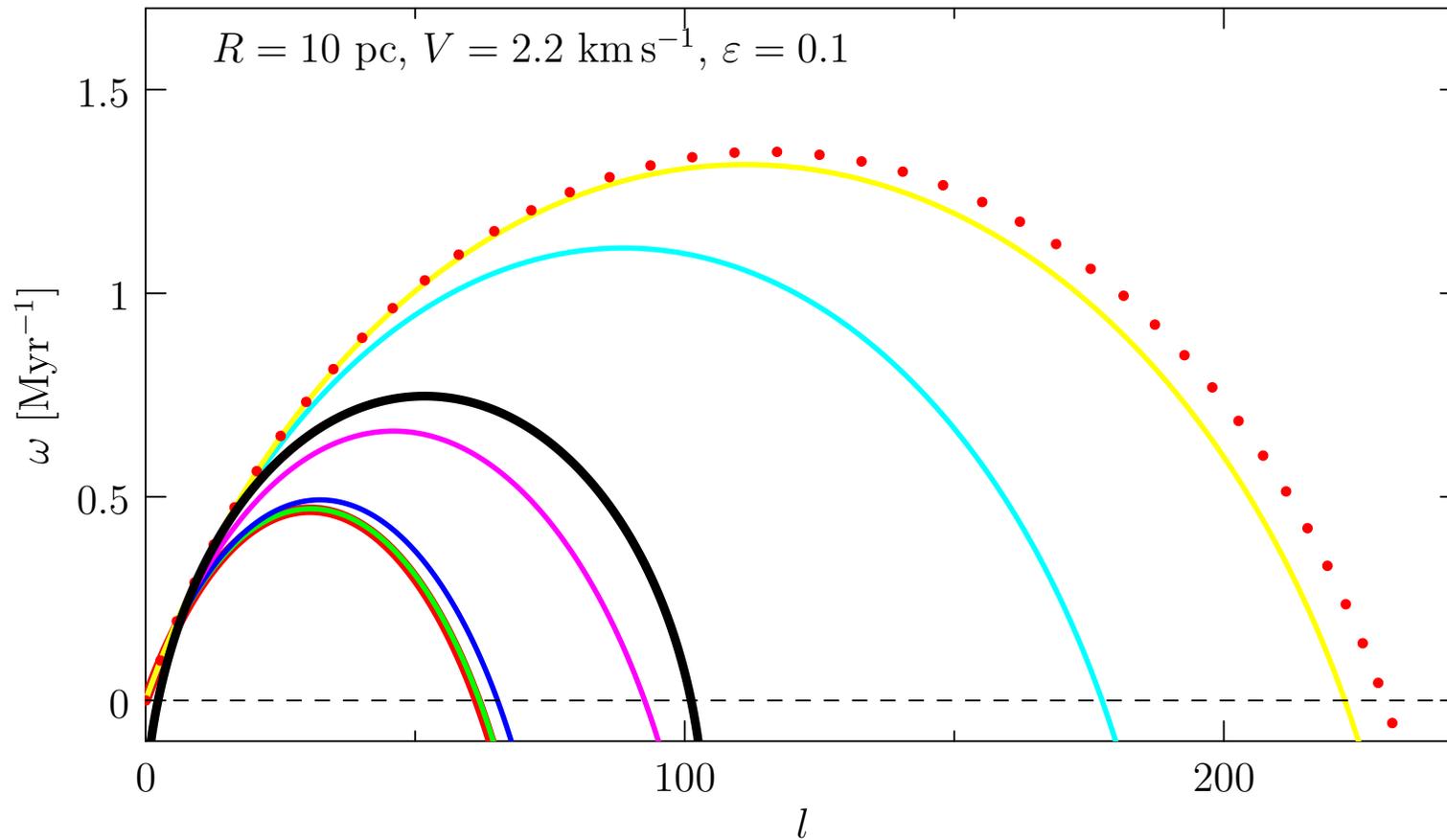
$$\omega_\epsilon = -\frac{V}{2R\epsilon} + \underbrace{\left\{ \frac{V^2}{4R^2\epsilon^2} + \frac{3G\Sigma_0 l}{4R\epsilon} \left[ \frac{\cos^{-1} \beta}{(1-\beta^2)^{3/2}} - \frac{\beta}{1-\beta^2} \right] \right\}}_{\text{gravity}} + \underbrace{\left\{ \frac{1 - P_{\text{ext}} c_s^2 l^2}{3\pi^2 R^2 \epsilon (2P_{\text{ext}} + \pi G \Sigma_0^2)} - \frac{5c_s^2 l^2}{2\pi^2 R^2 \epsilon} \right\}}_{\text{external pres.}}^{1/2} \cdot \underbrace{\left\{ \frac{5c_s^2 l^2}{2\pi^2 R^2 \epsilon} \right\}}_{\text{internal pres.}}^{1/2}.$$

Conf. to thin shell:

$$\omega_{\text{thin}} = -\frac{3V}{2R} + \left( \underbrace{\frac{V^2}{4R^2}}_{\text{stretching}} + \underbrace{\frac{2\pi G \Sigma_0 l}{R}}_{\text{gravity}} \underbrace{\frac{c_s^2 l^2}{R^2}}_{\text{internal pres.}} \right)^{1/2}$$

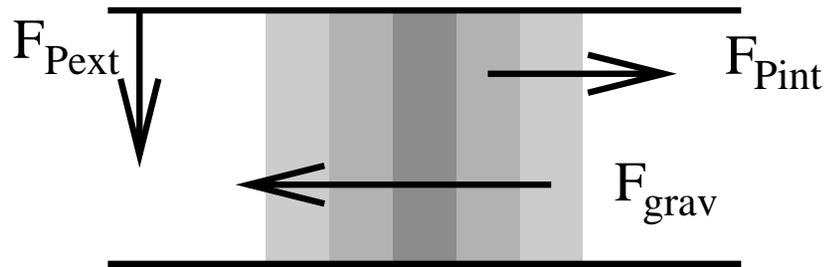
- geometry factor at gravity term
- external pressure term
- dependence on  $\epsilon$  - shrink fraction

# Pressure assisted gravitational instability

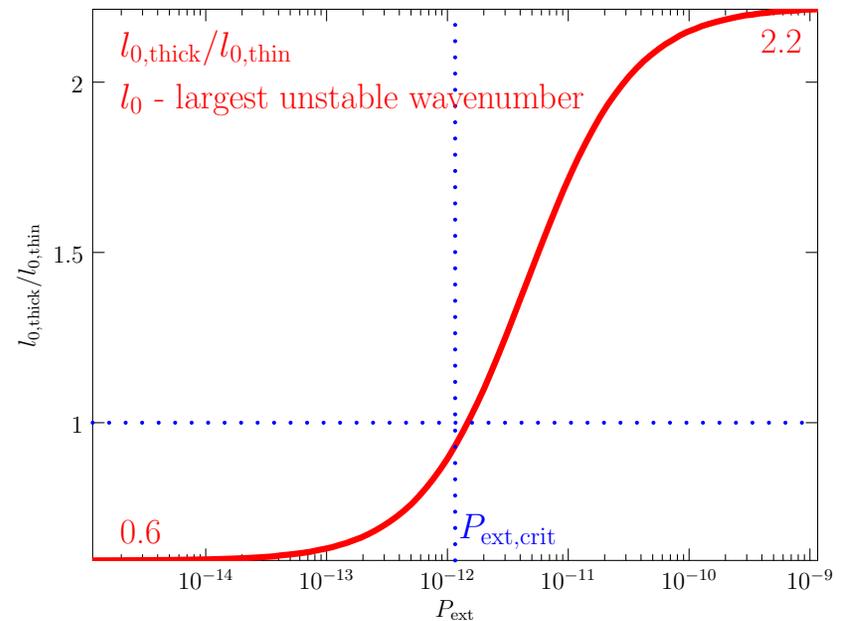
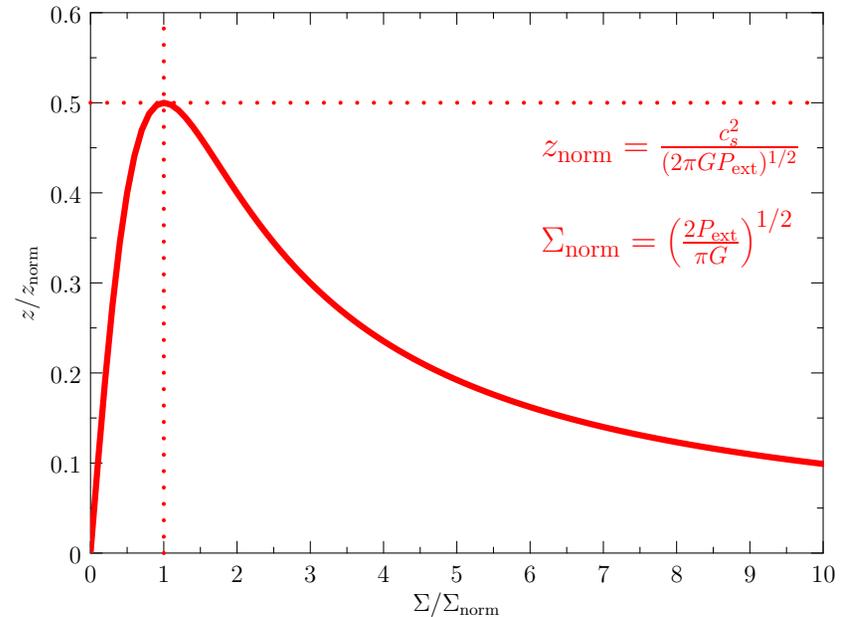


# Critical external pressure: $\omega_\epsilon \sim \omega_{\text{thin}}$

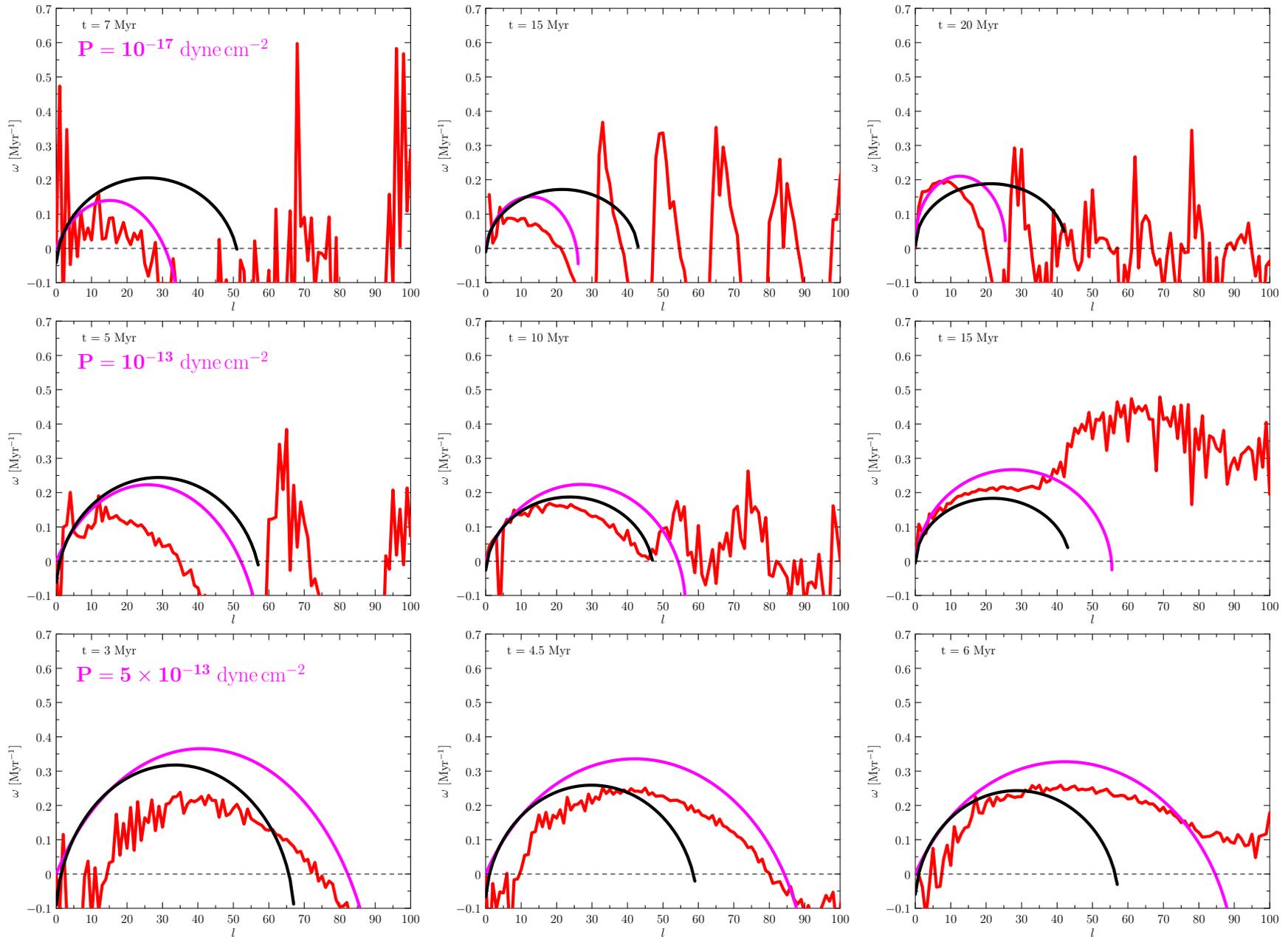
- $\omega_\epsilon \sim \omega_{\text{thin}}$  for  $P_{\text{ext,crit}}$  for which  $\frac{dz}{d\Sigma} = 0$  (the shell thickness does not depend on the surface density)



thin shell approx.



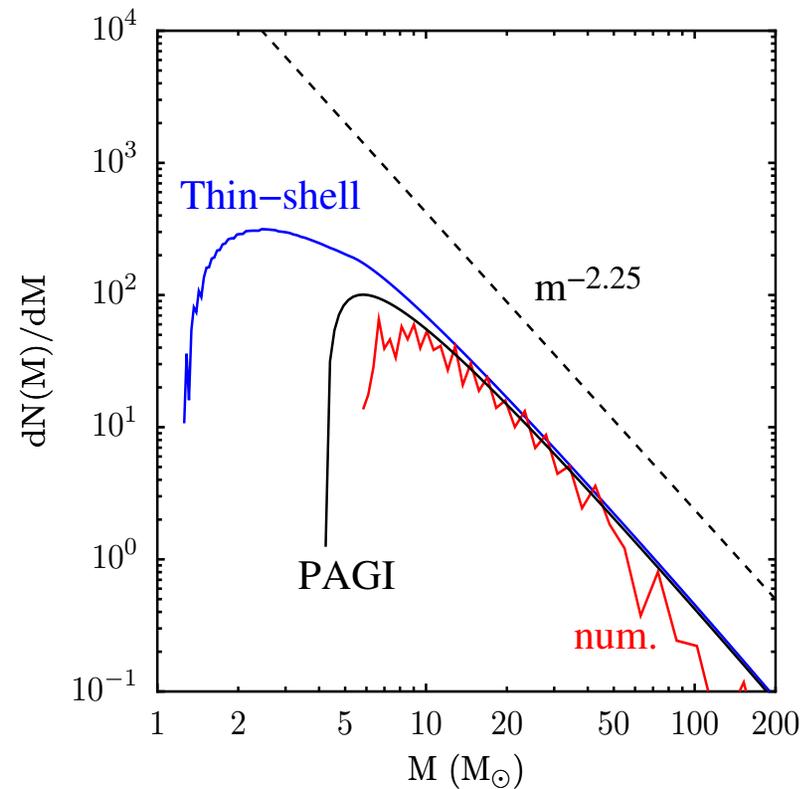
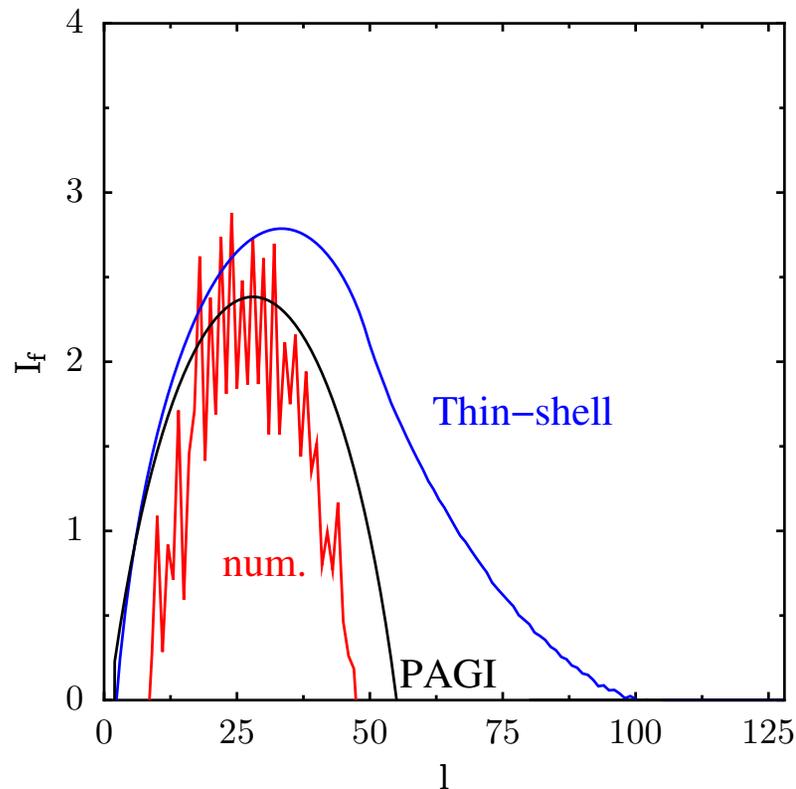
# PAGI vs. simulations



# Mass spectrum from Fragmentation Integral

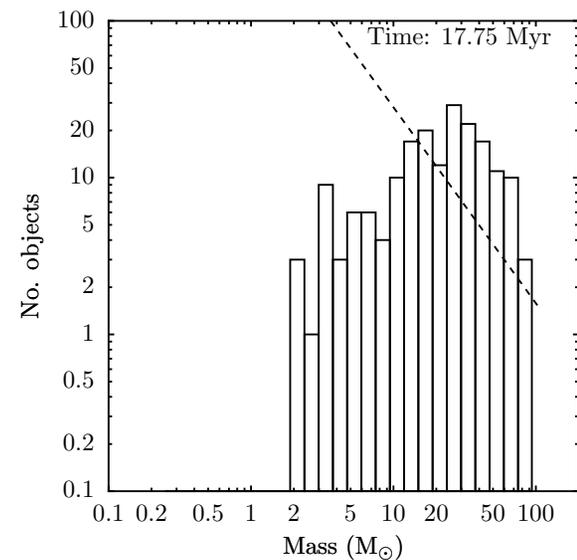
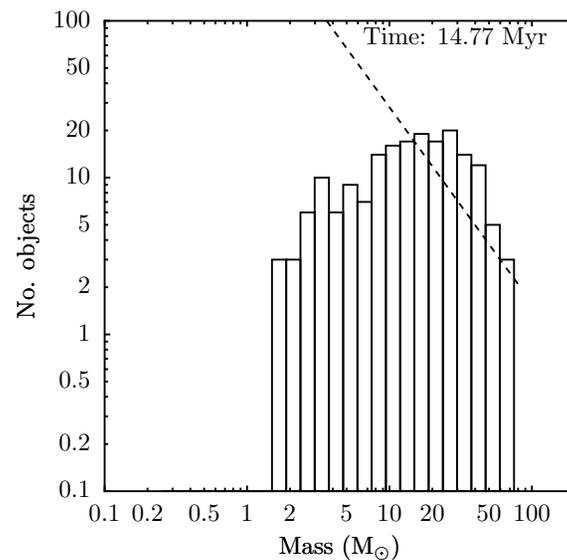
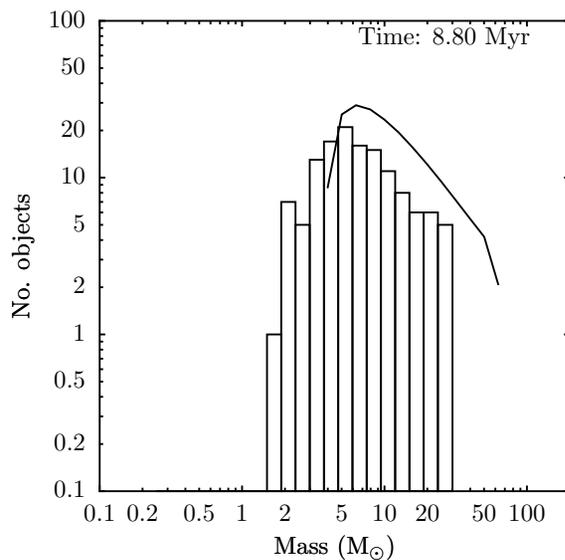
$$\frac{dN}{dm} \sim I_f(l, t) \times m^{-2}$$

where  $I_f(l, t) = \int \omega(l, t') dt'$  is the fragmentation integral



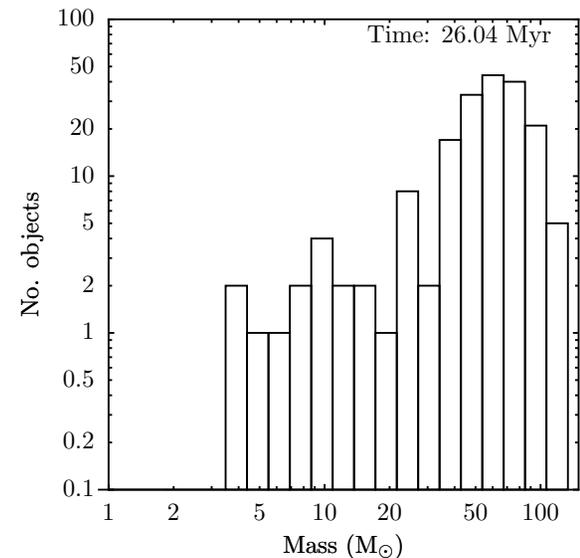
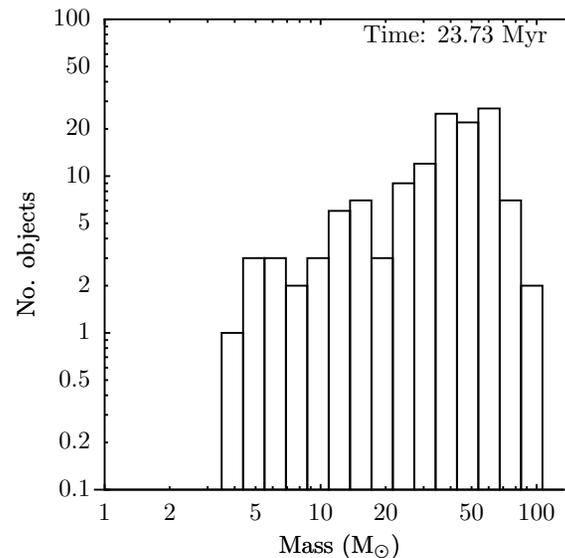
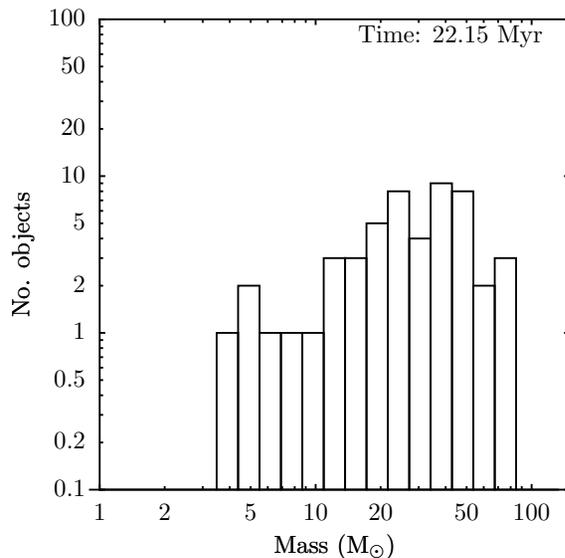
# Mass spectrum of clumps

- clumps identified by algorithm by Smith et al (2009) (similar to CLUMPFIND, but uses grav. potential instead of density)
- initially agrees well with mass spectrum obtain from the fragmentation integral
- later, it becomes more and more top-heavy



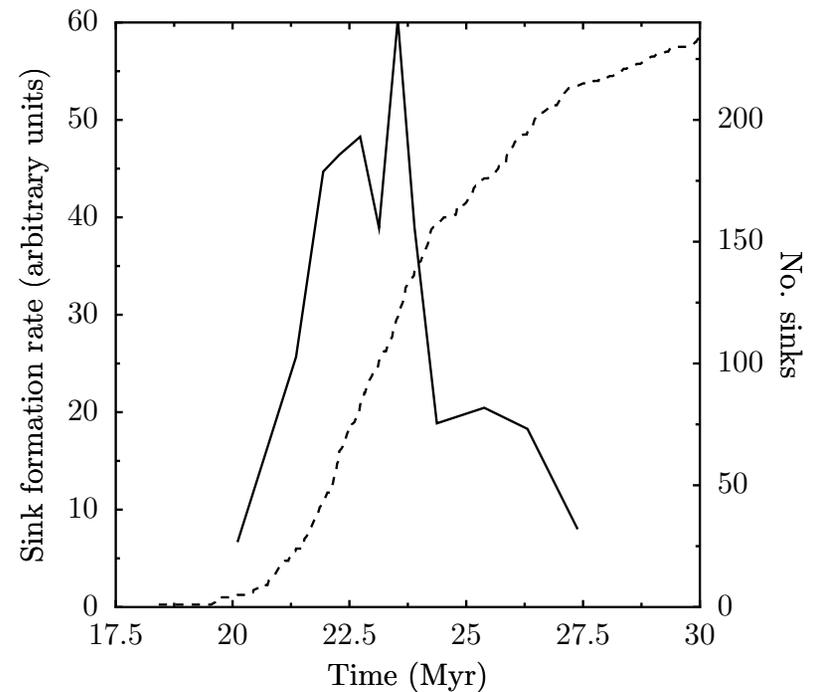
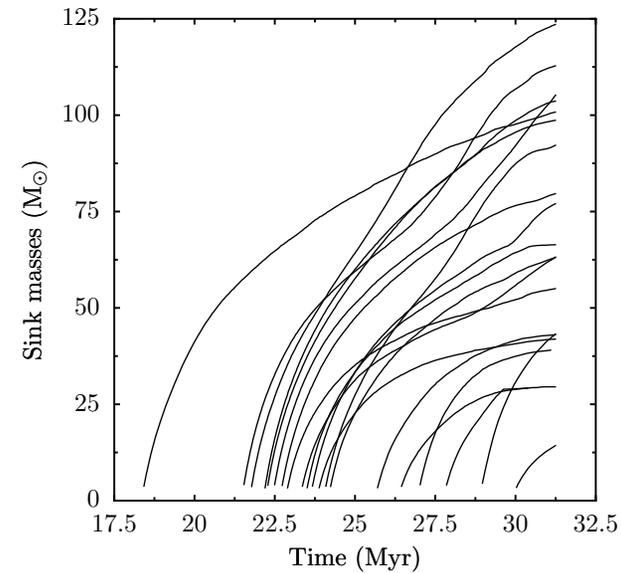
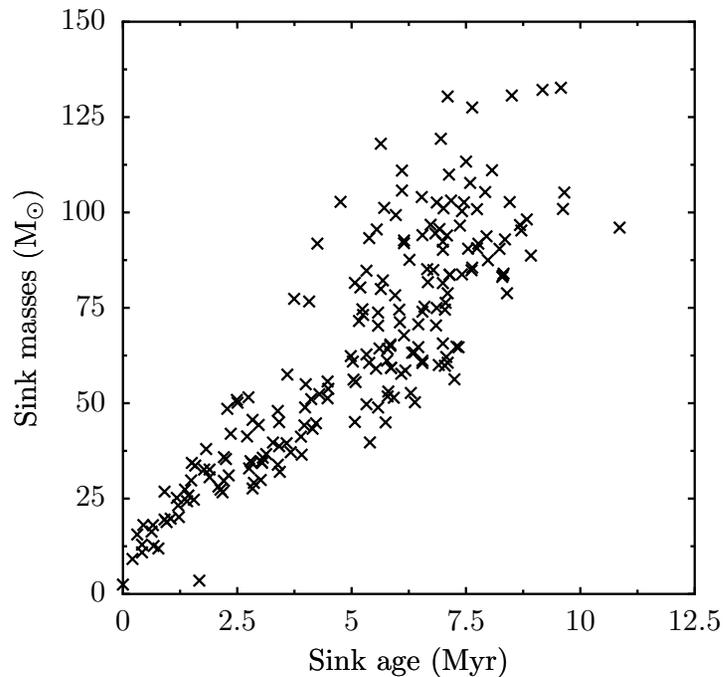
# Mass spectrum of sink particles

- sink particles created if gravitationally collapsing gas reaches resolution limits
- sink mass function is even more top-heavy
- peaks at  $M \sim 80 M_{\odot}$  which corresponds to  $l = 25$ , most unstable wavenumber at the time shell reaches maximum radius



# Oligarchic accretion

- period of high sink formation rate
- "oligarchs" from this period consume most of shell mass

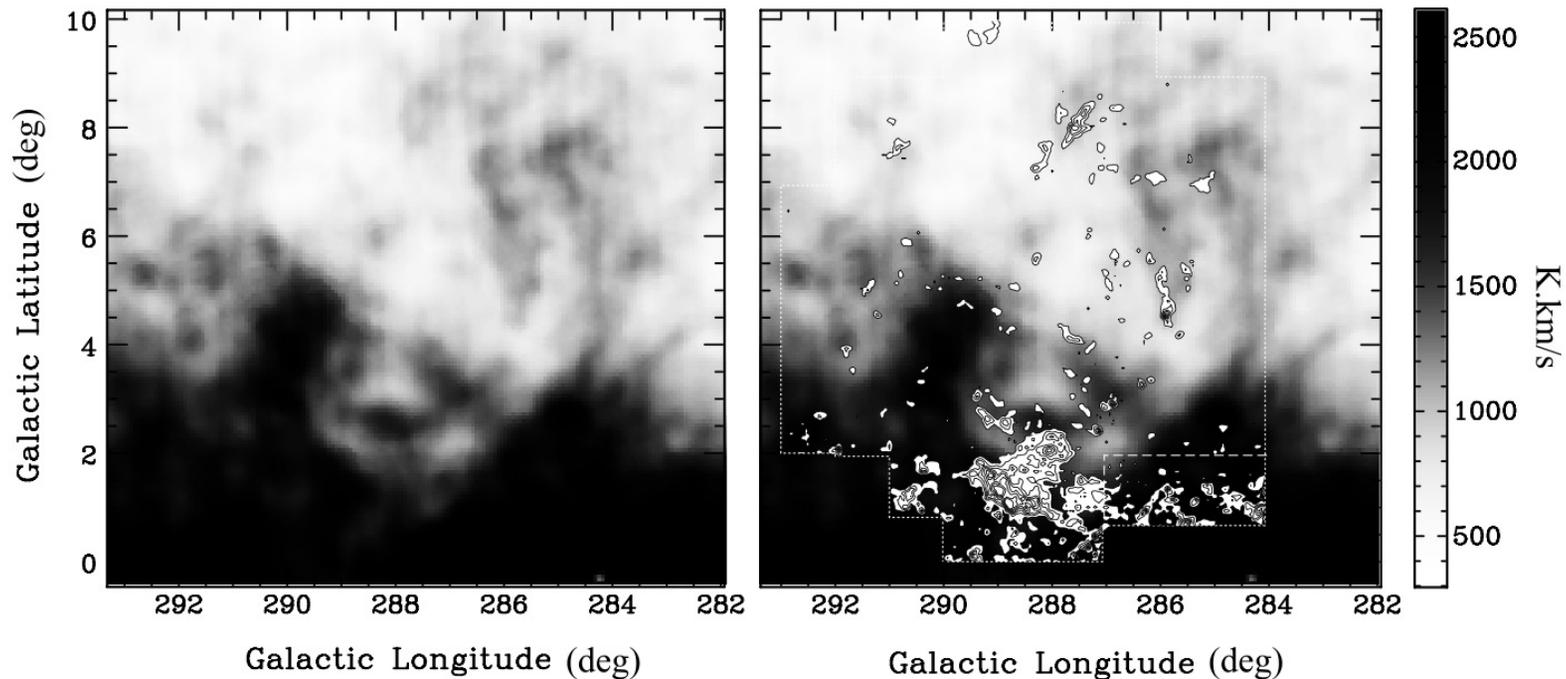


# Discussion

- PAGI gives systematically larger range of unstable wavelengths than simulations (?approximation of the UOS evolution by constant acceleration)
- $\omega$  only amplifies initial spectrum (given by the turbulence, . . . )
- we assume shell confined by thermal pressure from both sides, but instability can be different if the ram pressure confines the shell from outside (accreting shell)
- Vishniac and RT instabilities may have an impact
- Oligarchic accretion and the resulting top-heavy CMF appear because the gas reservoir to form new cores is depleted (shell does not accrete); Klessen & Burkert (2000) observe similar process in evolution of turbulent protocluster if SFE is high

# Carina Flare Supershell observation

- PAGI predicts deficit of low mass fragments in low pres environments (and vice versa)
- Oligarchic accretion predicts top-heavy CMF for non-accreting shells
- we plan to use the APEX telescope (got 36 hours) to determine CMF of the Carina Flare supershell (extends  $\sim 450$  pc above the galactic plane)



# Conclusions

- excellent agreement between AMR and SPH, but disagreement with the thin shell approximation
- new instability (PAGI) and dispersion relation (fragment growth rate) for the thick shell embedded in the medium with non-zero pressure
- the PAGI dispersion relation depends on the external pressure, predicts range of unstable wavenumbers different than the one given by the thin shell approximation by factor of 0.6 and 2.2 for  $P_{\text{ext}} = 0$  and  $\infty$ , respectively
- the PAGI dispersion relation is similar to the thin shell one for the pressure for which the shell thickness locally does not depend on its surface density (maximum shell thickness)
- early stage: mass spectrum of clumps agrees well with analytical estimates (PAGI)
- later stage: oligarchic accretion leads to the top-heavy CMF

# References

- Dale, J. E., Wunsch, R., Whitworth, A., Palouš, J., 2009, MNRAS, 398, 1537
- Wunsch, R., Dale, J. E., Palouš, J., Whitworth, A., 2010, MNRAS, 407, 1963
- Dale, J. E., Wunsch, R., Smith, R. J., Whitworth, A., Palouš, J., 2010, MNRAS, in press
- Boyd & Whitworth, 2005, A&A, 430, 1059
- Churchwell, E., et al, 2006, ApJ, 649, 759
- Deharveng, L., Lefloch, B.; Zavagno, A.; Caplan, J.; Whitworth, A. P.; Nadeau, D.; Martín, S. 2003, A&A, 408, L25
- Ehlerová, S., Palouš, J., 2005, A&A, 437, 101
- Elmegreen, B. G., Lada, C. J., 1977, ApJ, 214, 725
- Elmegreen, B. E., 1994, ApJ, 427, 384
- Garcia-Segura, G., Franco, J., 1996, ApJ, 496, 171
- Klessen, R., Burkert, A., 2000, ApJS, 128, 287
- Parker, E. N., 1966, ApJ, 145, 811
- Sidorin, V. 2008, Master thesis, Charles University
- Smith, R. J., Clark, P. C., Bonnell, I. A., 2009, MNRAS, 396, 830
- Vishniac, E. T., 1983, ApJ, 274, 152
- Wardle, M. 1990, MNRAS, 246, 98