Thickness matters
in self-gravitating expanding shells

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Outline:

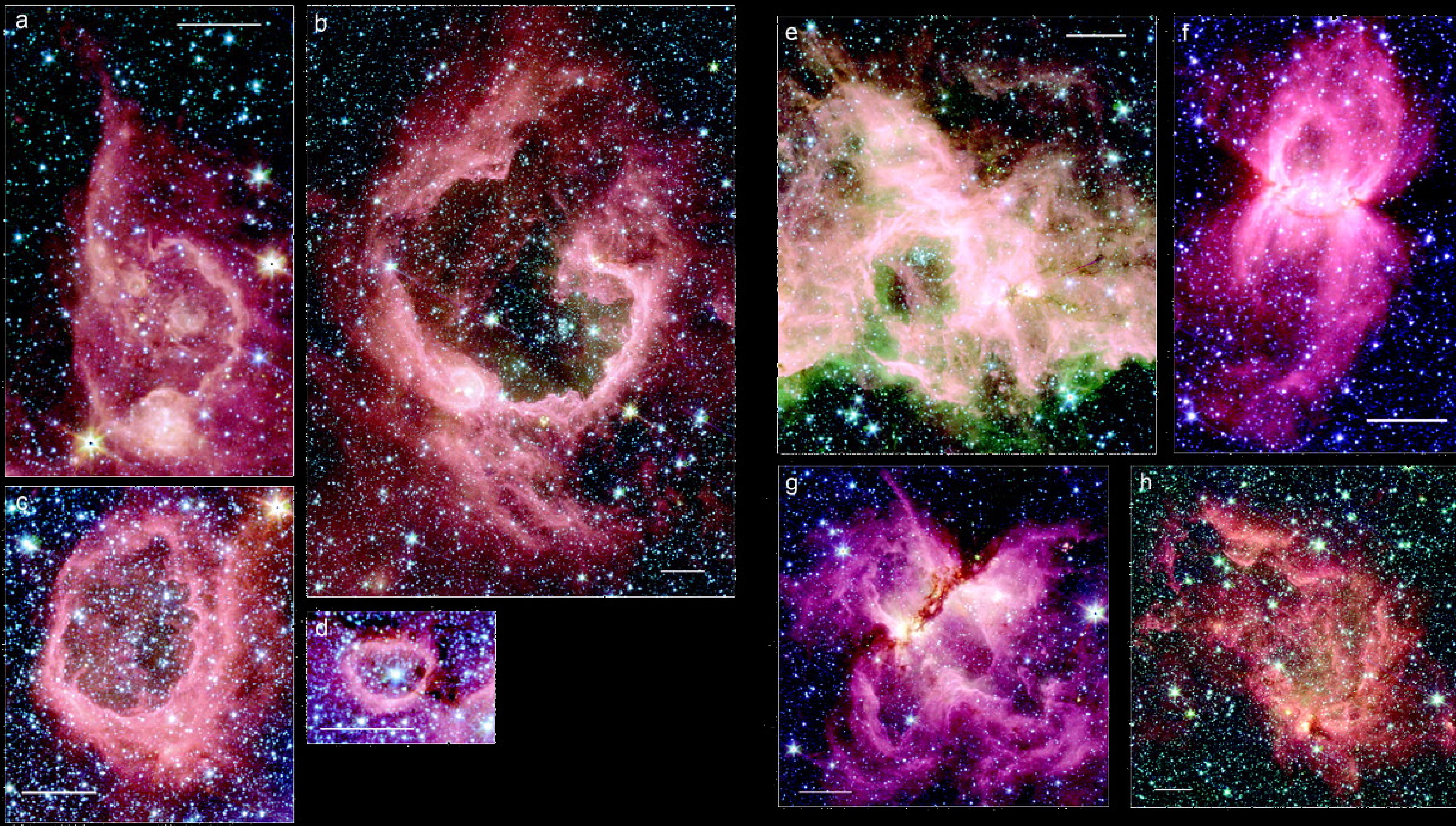
1. **Observational evidence**
2. **Collect and collapse, gravitational instability**
3. **AMR vs. SPH simulations vs. theory**
4. **Pressure assisted gravitational instability**

Galactic bubble N107, Credit: Churchwell et al. (2006), Spitzer, GLIMPSE, IRAC, 8μm cont.


THE INFRARED MILKY WAY: GLIMPSE (3.6–8.0 microns)
SPITZER
SPACE TELESCOPE
Galactic Latitude (degrees)
Galactic Longitude (degrees)
Poster designed by Thomas Robitaille and Robert Hurt

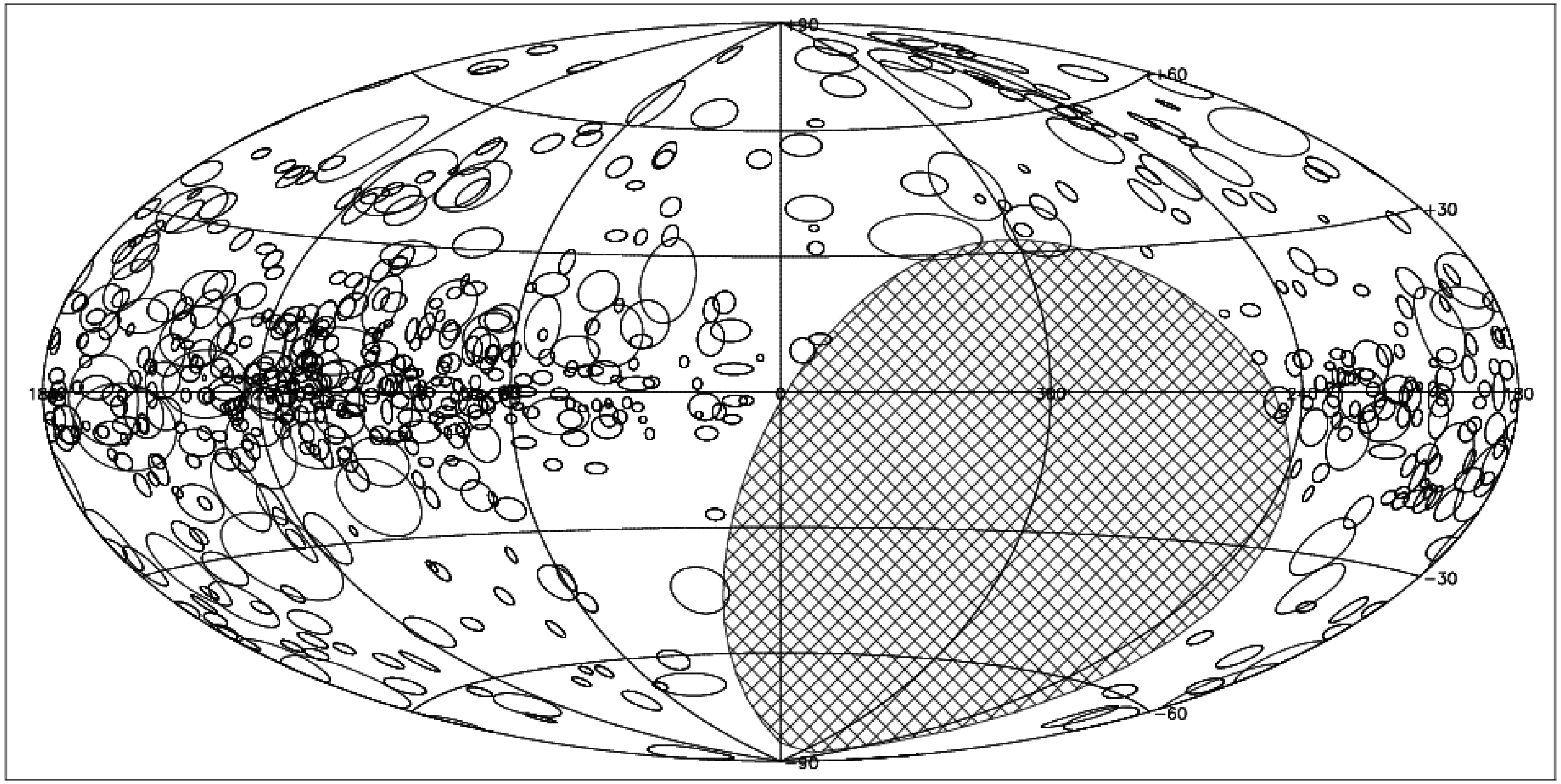
The bubbling galactic disk

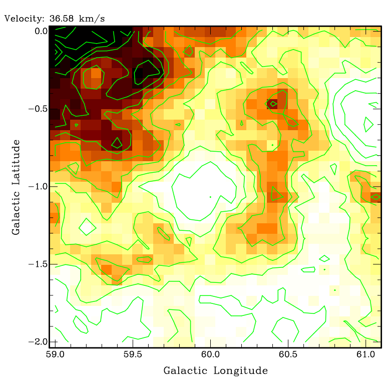
(Churchwell et al., 2006)

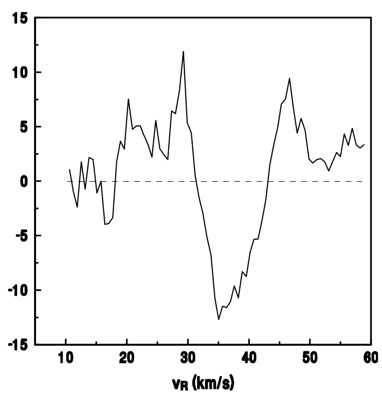


Spitzer bands: 4.5μm (blue), 5.8μm (green), 8μm (red)

Bubbles are everywhere







automatic search for HI shells in Leiden-Dwingeloo survey
$\rightarrow$ statistical study of 300 HI shells in 2nd quadrant
(Ehlerová et al. 2005)

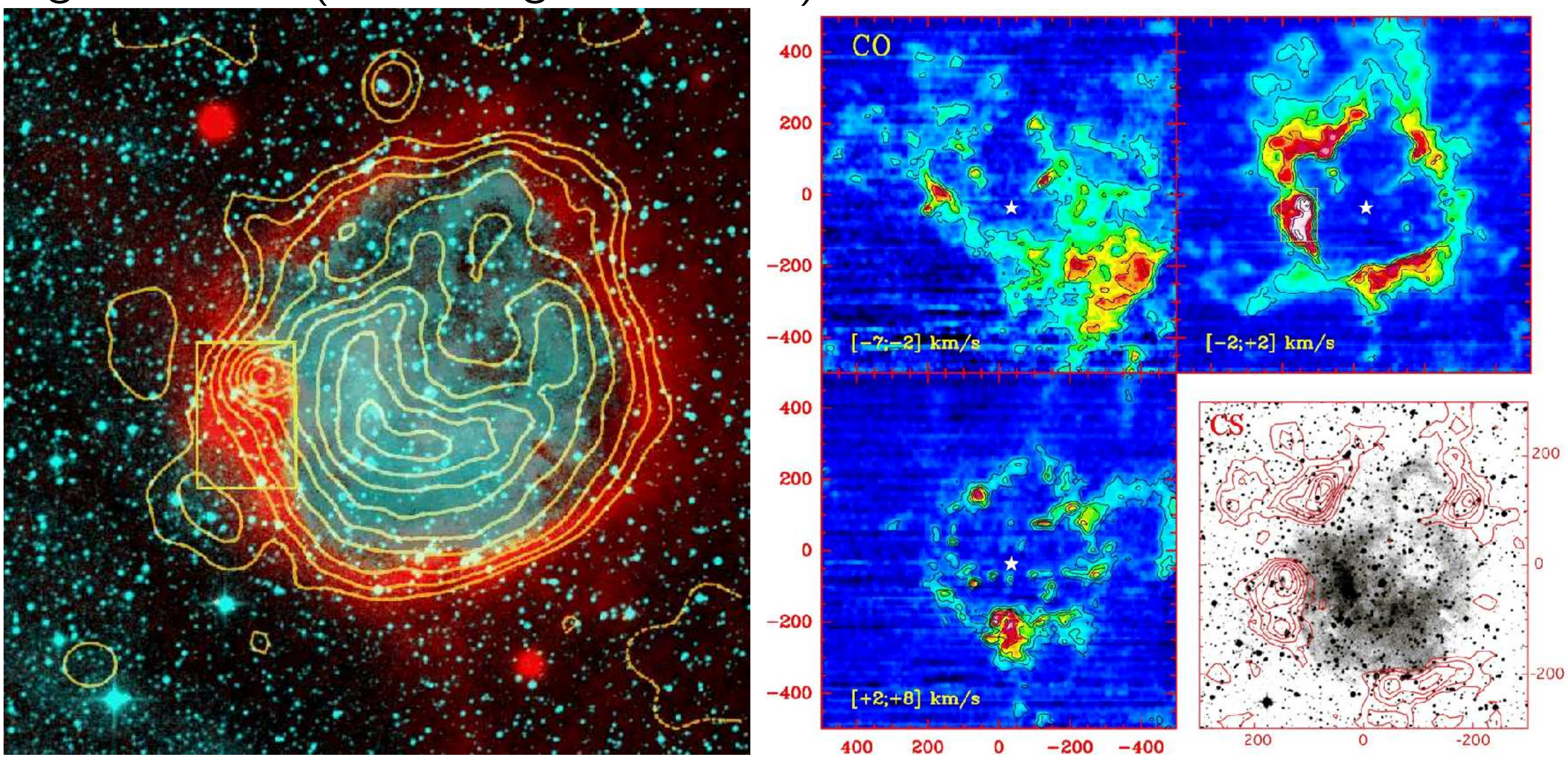
Bubble properties

- radii: $r \sim 10$ pc - 2 kpc
- expansion velocities: $v_{\rm exp} \sim 5 - 30$ km/s
- formation energy: $E \sim 10^{51} - 10^{53}$ erg
- observed in many wavelengths: HI (IGALFA, . . .), IR (Spitzer/GLIMPSE,. . .), Hα (WHAM), mm (CO lines), radio cont., X-rays
- observed in MW and many nearby galaxies (LMC, SMC, M31, M33, IC10 . . .)
- origin: OB stars (fossils of expanding HII regions), encounters with HVC or dwarf galaxy, GRB, turbulence and instabilities in ISM
- implications: source of information about ISM, Disk-halo connection, **Triggered star formation (Collect and Collapse)**

Collect and collapse

(Elmegreen & Lada, 1977)

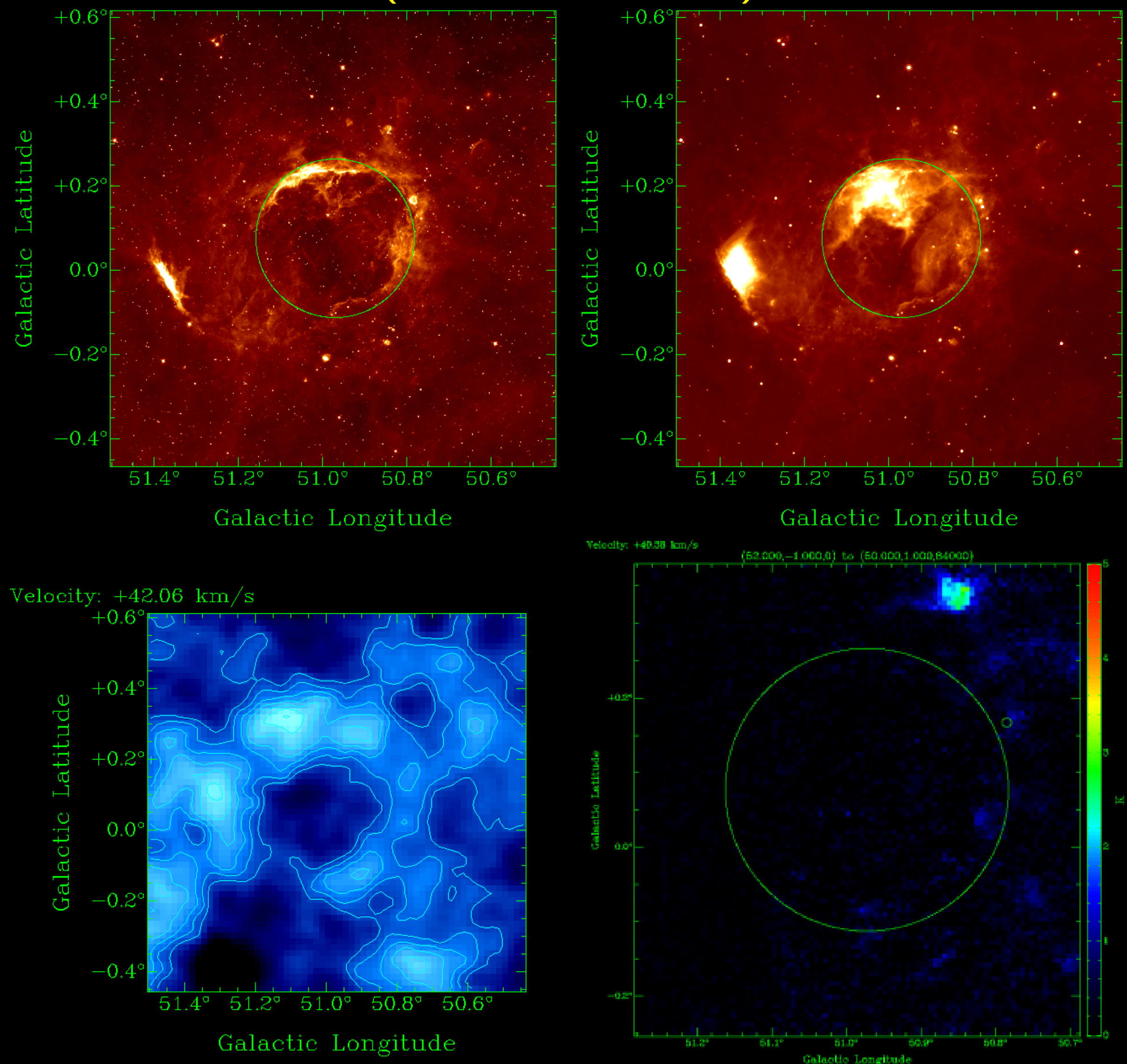
HII region Sh 104 (Deharveng et al., 2003)



Left: contours (thermal radio continuum 1.46 GHz), red (mid-IR emmision - PAHs), turquoise (Hα - ionized gas)
Right: CO molecilar line at different velocities

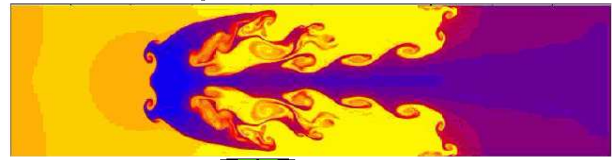
Multiwavelength study of N107

(Sidorin, 2008)

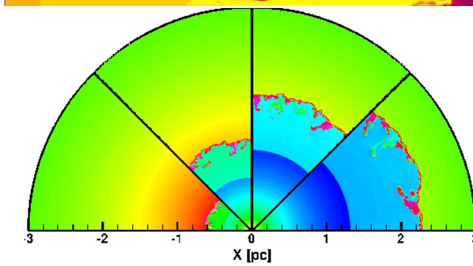


Expanding shell instabilities

- gravitational instability (long time-scale)
- Rayleigh-Taylor instability

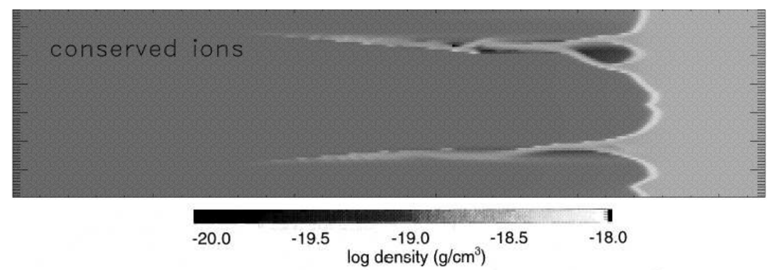


- Vishniac instability
  (Vishniac, 1983; 1994)



- magnetic field:
  Parker inst. (Parker 1966)

  Wardle inst. (Wardle, 1990)



- ionized shell instability
  (Garcia-Segura & Franco, 1996)

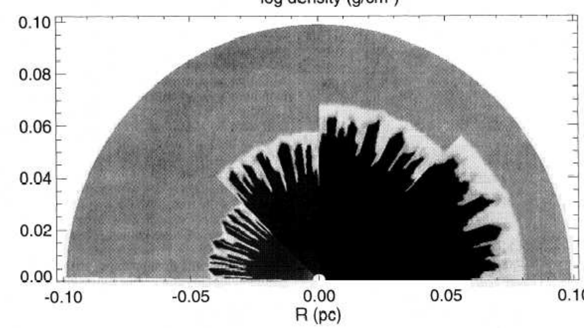
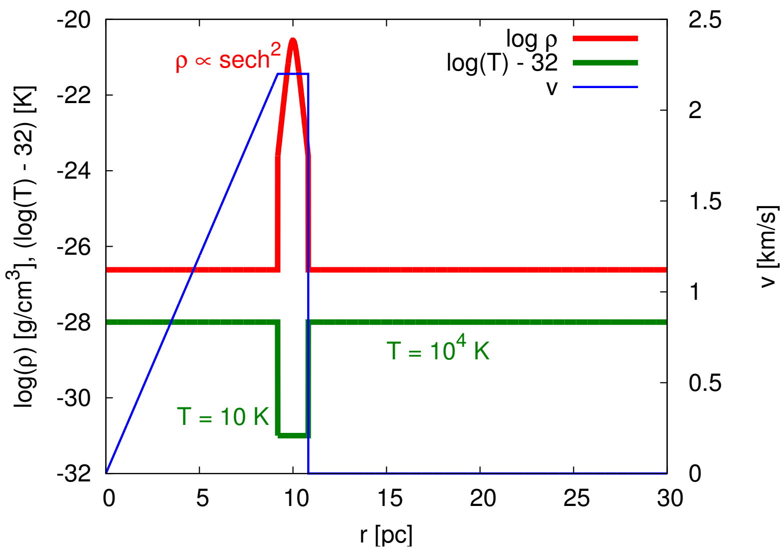
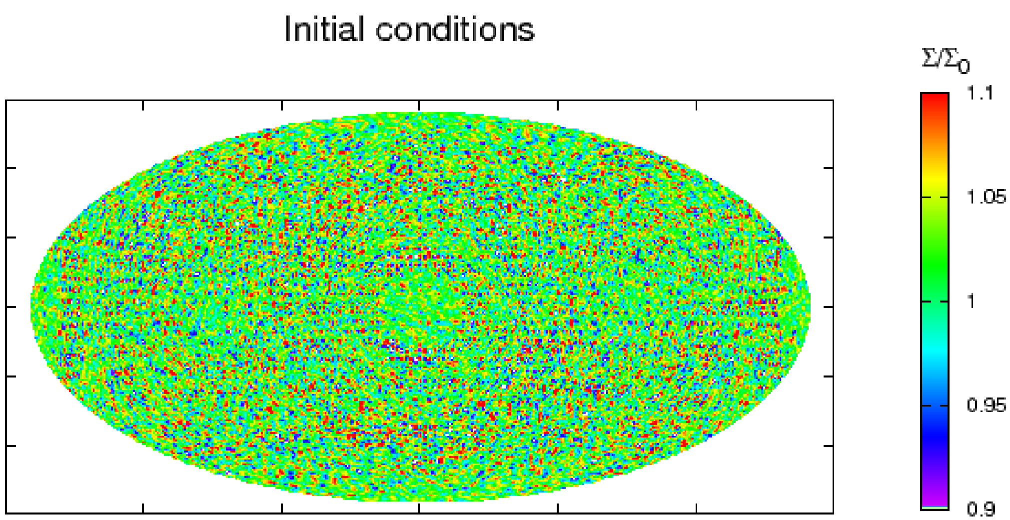


FIG. 6.—Evolution of the I-S front instability for a case with constant density (model UC32). The cooling cutoff is 10^3 K, and the time step is 5×10^3 yr. The ambient medium has $n_0 = 10^5$ cm^{-3} and $T_0 = 100$ K, and the stellar flux is $F_* = 10^{48}$ s^{-1}. This model is equivalent to the very early stages of model S32, shown in Fig. 8.

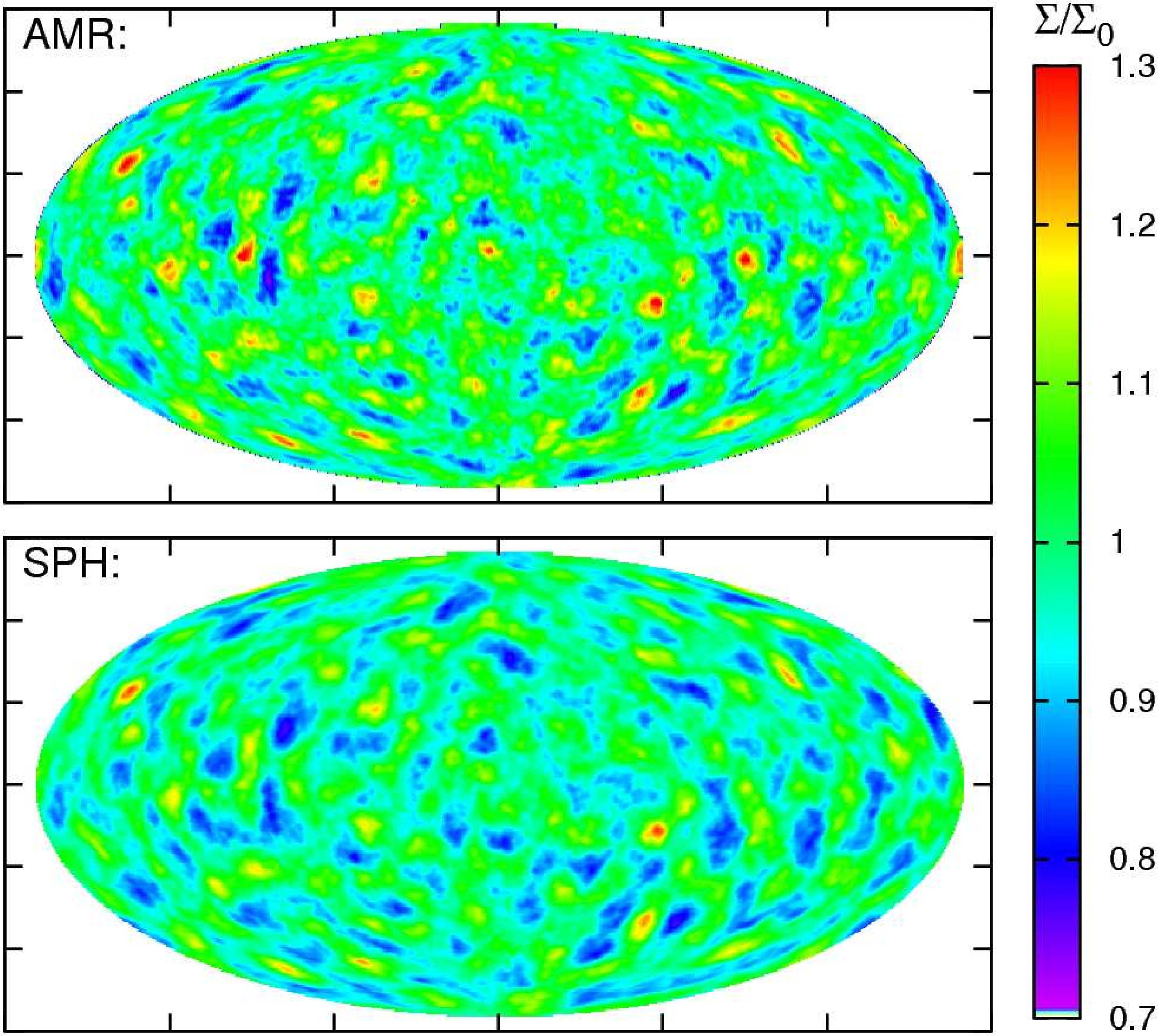
Simulation setup

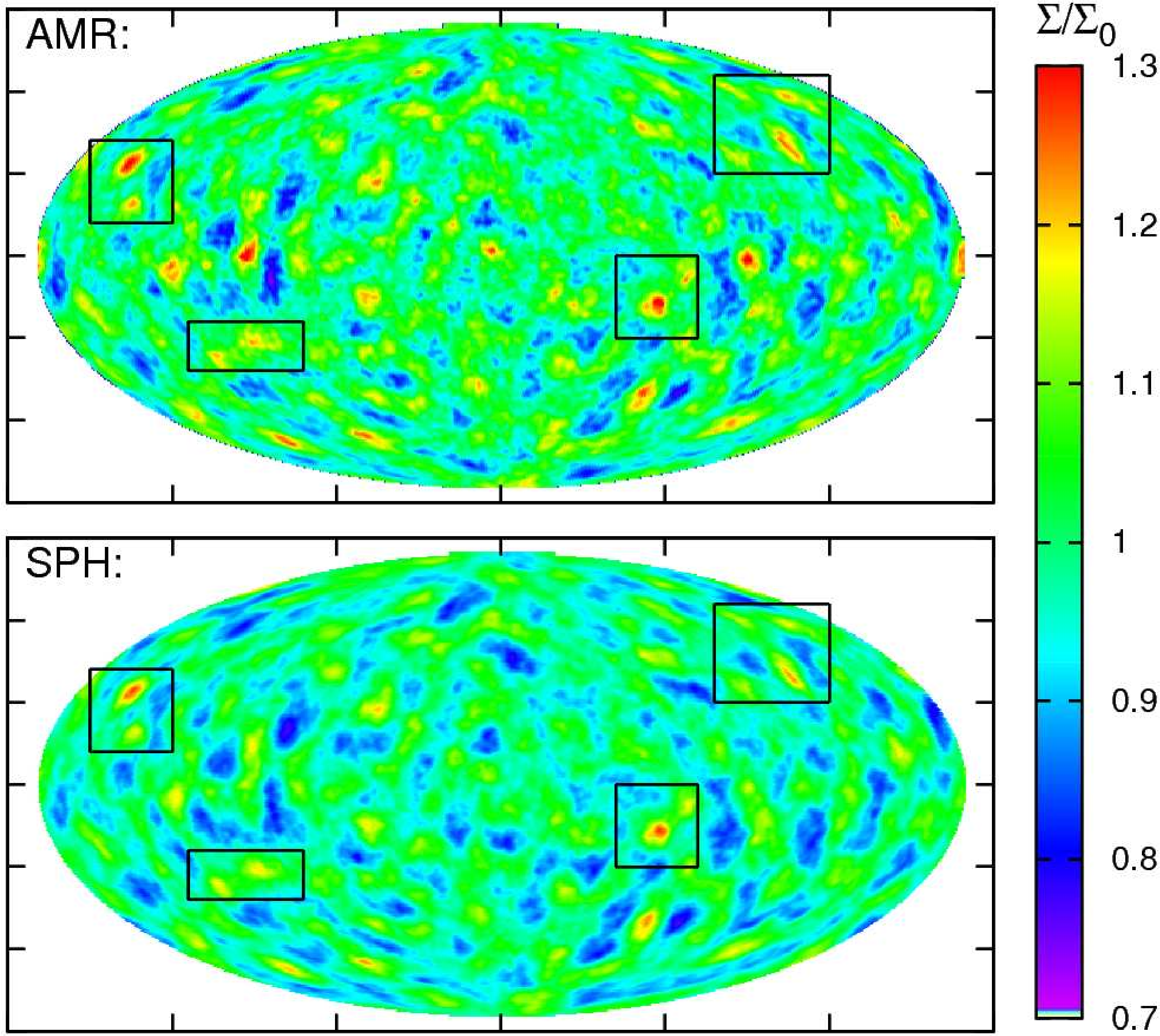
- extremely simplified model to avoid instabilities other than the gravitational one (RT, Vishniac)
- ballistic shell (in a free fall) embedded in a rarefied medium





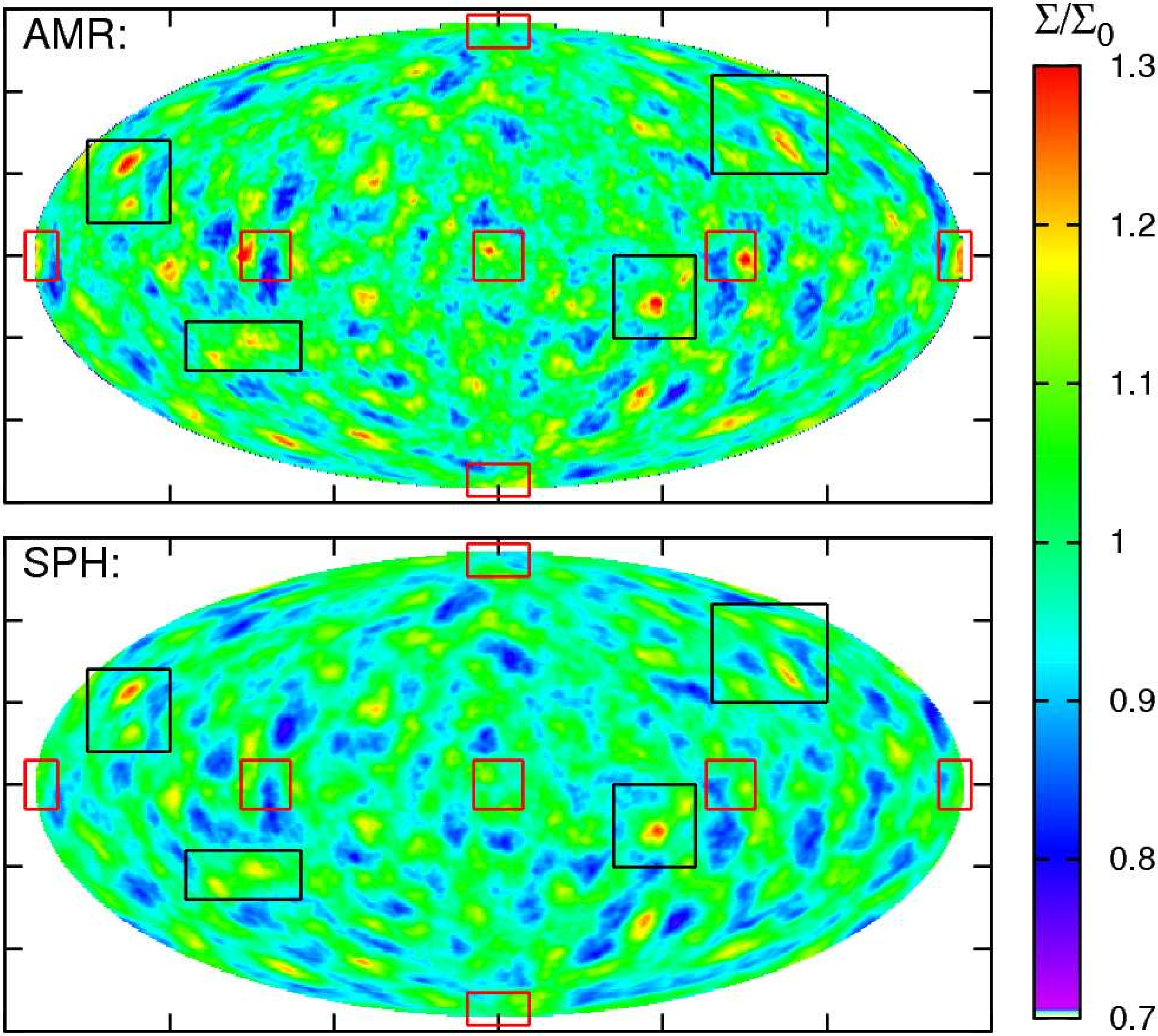
$M_{\mathrm{shell}} = 2 \times 10^4$ M$_\odot$
$T_{\mathrm{shell}} = 10$ K
$R_{\mathrm{shell},0} = 10$ pc
$V_{\mathrm{shell},0} = 2.2$ km s^{-1}
$R_{\mathrm{shell,max}} = 23$ pc
$P_{\mathrm{ext}} = 10^{-17}$, 10^{-13}
or 5×10^{-13} dyne cm^{-2}


AMR:
SPH:
Σ/Σ_0
1.3
1.2
1.1
1
0.9
0.8
0.7

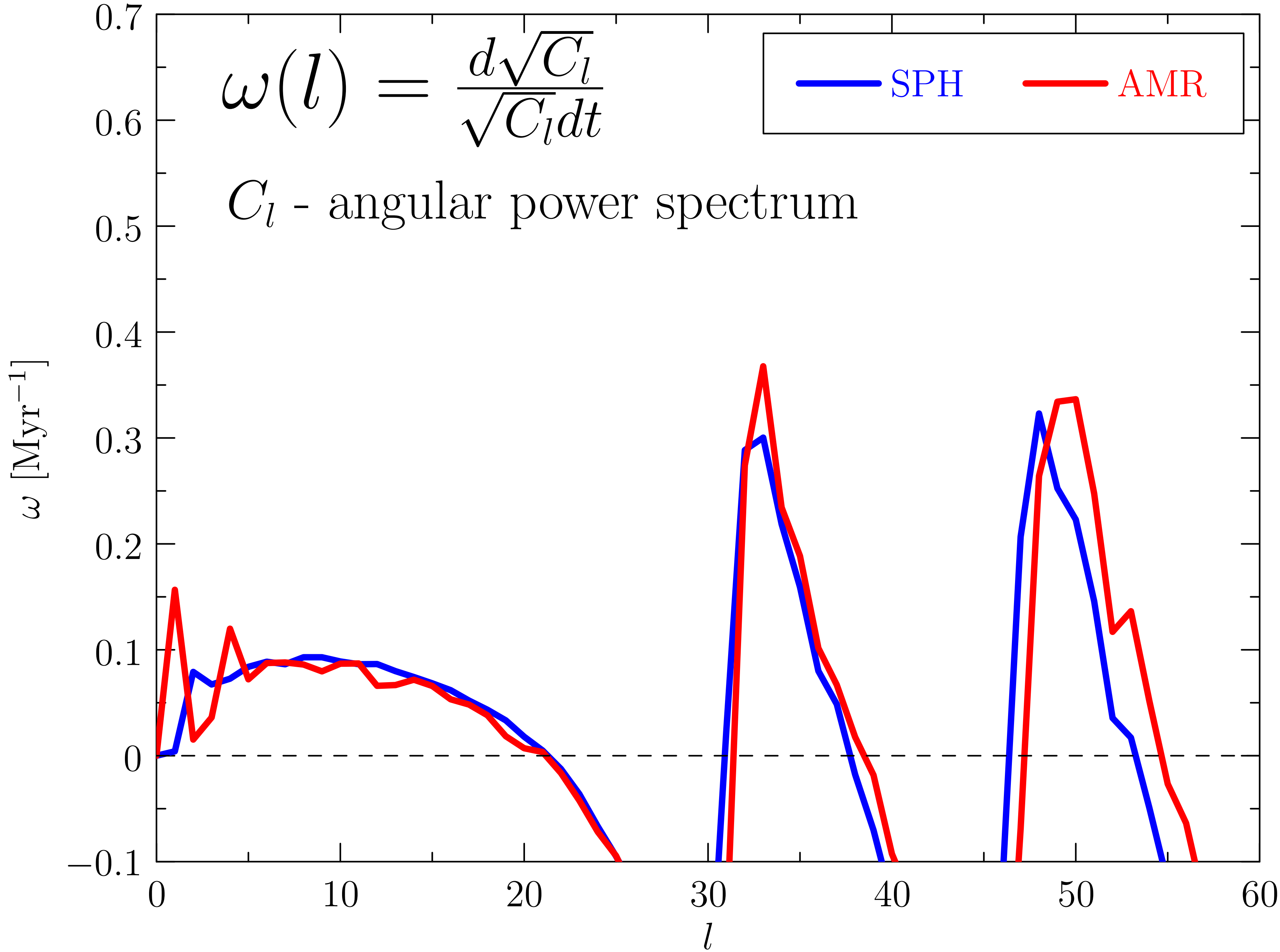


AMR:
SPH:
Σ/Σ0
1.3
1.2
1.1
1
0.9
0.8
0.7

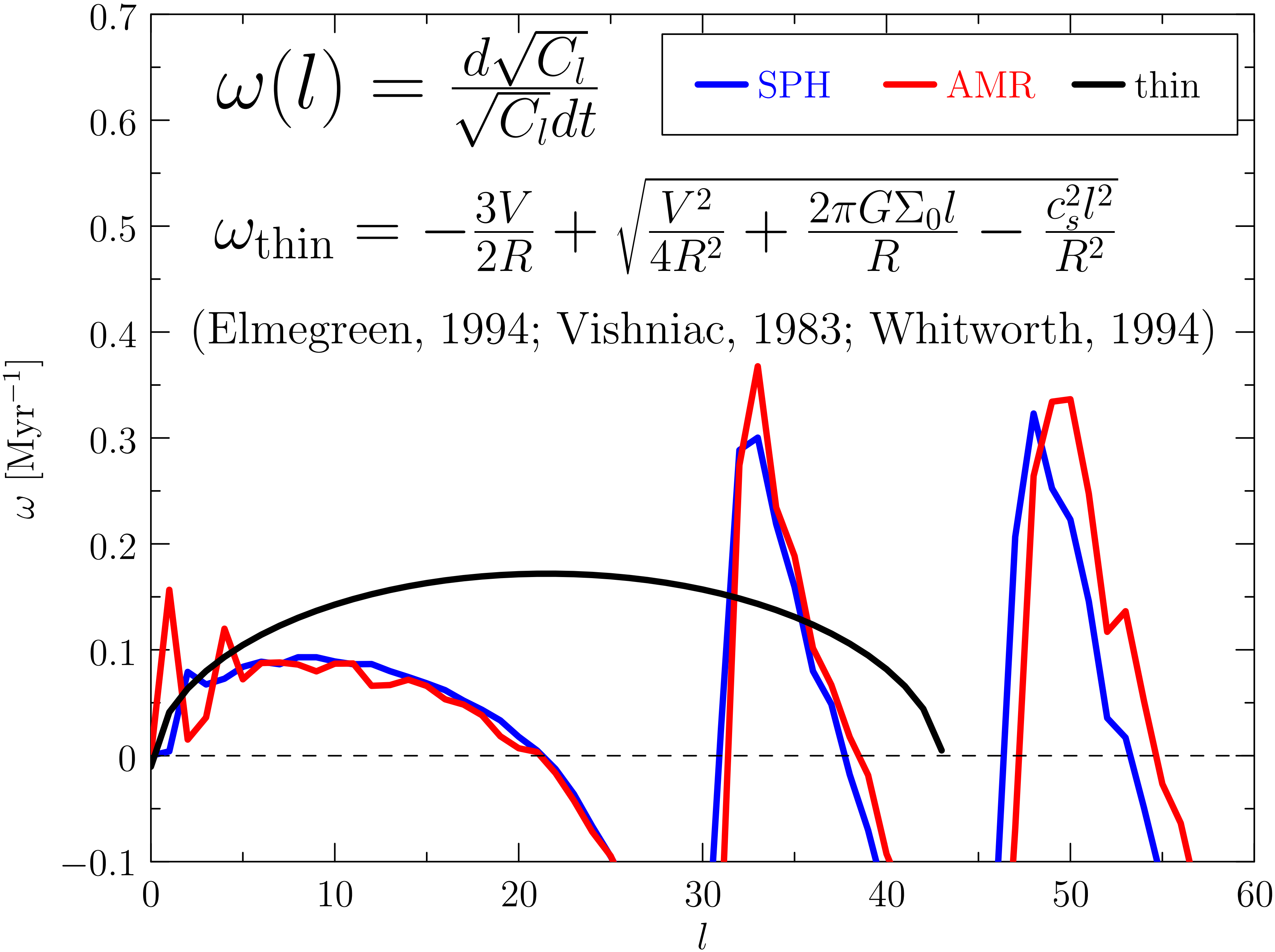
Find six differences!



AMR:
SPH:
Σ/Σ0
1.3
1.2
1.1
1
0.9
0.8
0.7



$\omega(l) = \frac{d\sqrt{C_l}}{\sqrt{C_l}dt}$
C_l - angular power spectrum
SPH
AMR
ω [Myr^{-1}]
l
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
0
10
20
30
40
50
60



$\omega(l) = \frac{d\sqrt{C_l}}{\sqrt{C_l}dt}$
SPH
AMR
thin
$\omega_{\rm thin} = -\frac{3V}{2R} + \sqrt{\frac{V^2}{4R^2} + \frac{2\pi G\Sigma_0 l}{R} - \frac{c_s^2 l^2}{R^2}}$
(Elmegreen, 1994; Vishniac, 1983; Whitworth, 1994)
ω [Myr^{-1}]
l
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
0
10
20
30
40
50
60

Gravitational instability of thin shell

(Vishniac, 1983; Whitworh et al. 1994; Elmegreen, 1994)

pressure, gravity, stretching, accretion

$$\Sigma_0 R \frac{\partial \mathbf{\Omega}}{\partial t} = -c_s^2 \nabla \Sigma_1 + \Sigma_0 \nabla \Phi_1 - \Sigma_0 \mathbf{\Omega} V - \cancel{3 \Sigma_0 \mathbf{\Omega} V}$$

$$\frac{\partial \Sigma_1}{\partial t} = -\Sigma_0 R \nabla_{\mathrm{T}} \cdot \mathbf{\Omega} - 2 \Sigma_1 \frac{V}{R}$$

stretching

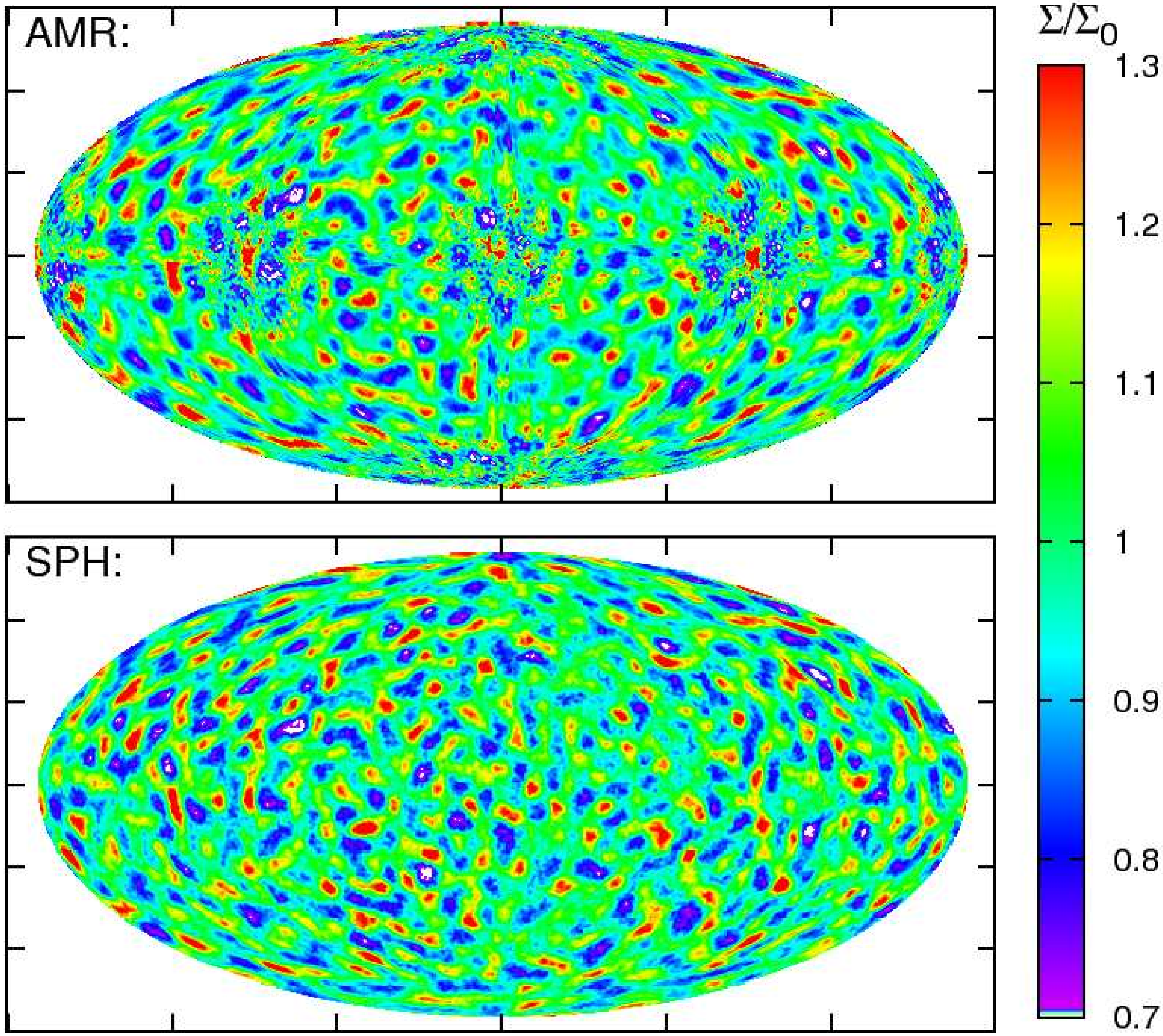
$$\nabla^2 \Phi_1 = 4 \pi G \Sigma_1 \delta(r - R)$$

$\rightarrow$ linearization, perturbations inserted

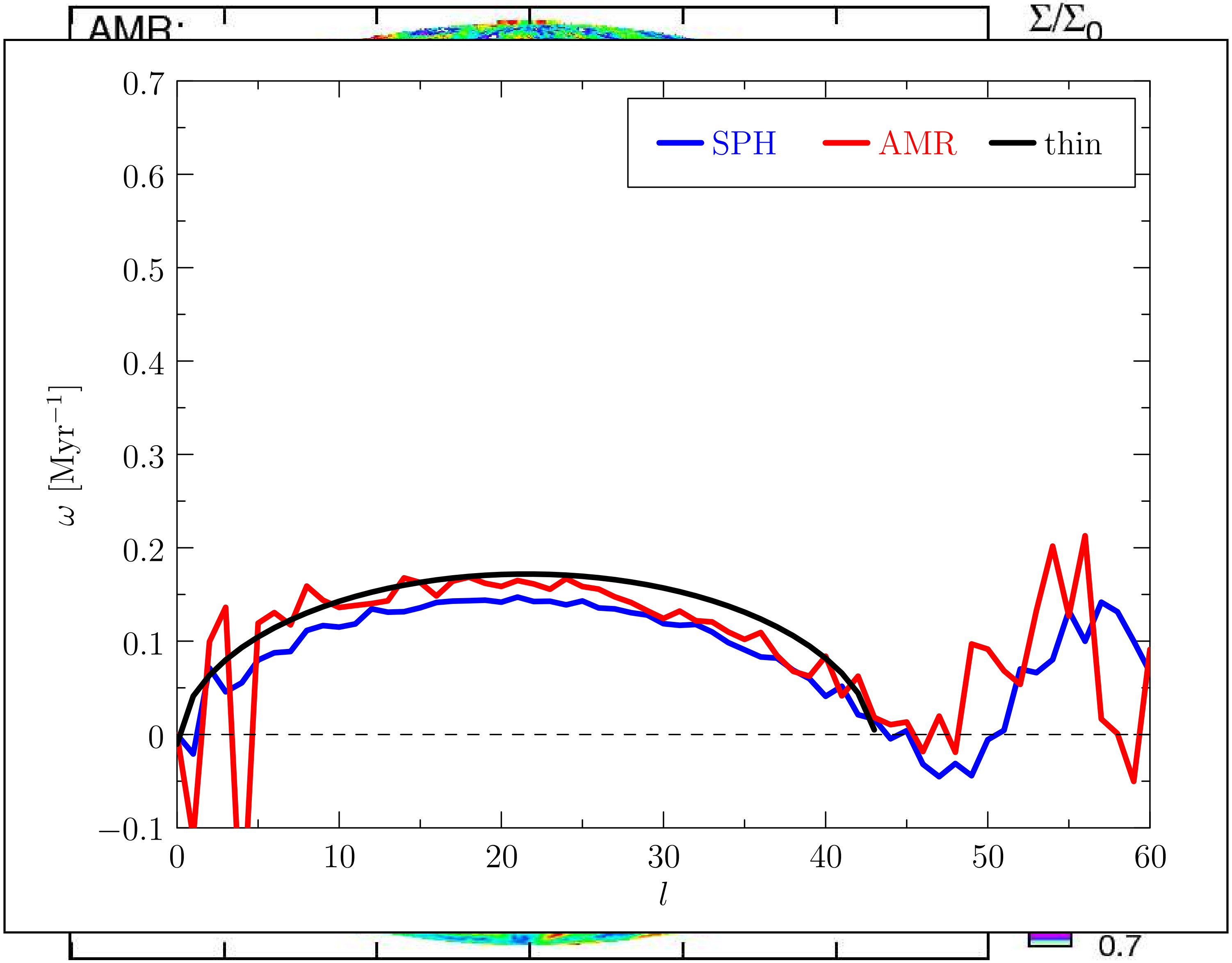
mode with dimensionless wavenumber $l = kR$ studied:

$$\omega_{\mathrm{thin}}(l) = -\frac{3V}{R} + \sqrt{\frac{V^2}{R^2} + \frac{2\pi G \Sigma_0 l}{R} - \frac{c_s^2 l^2}{R^2}}$$

High pressure ambient medium

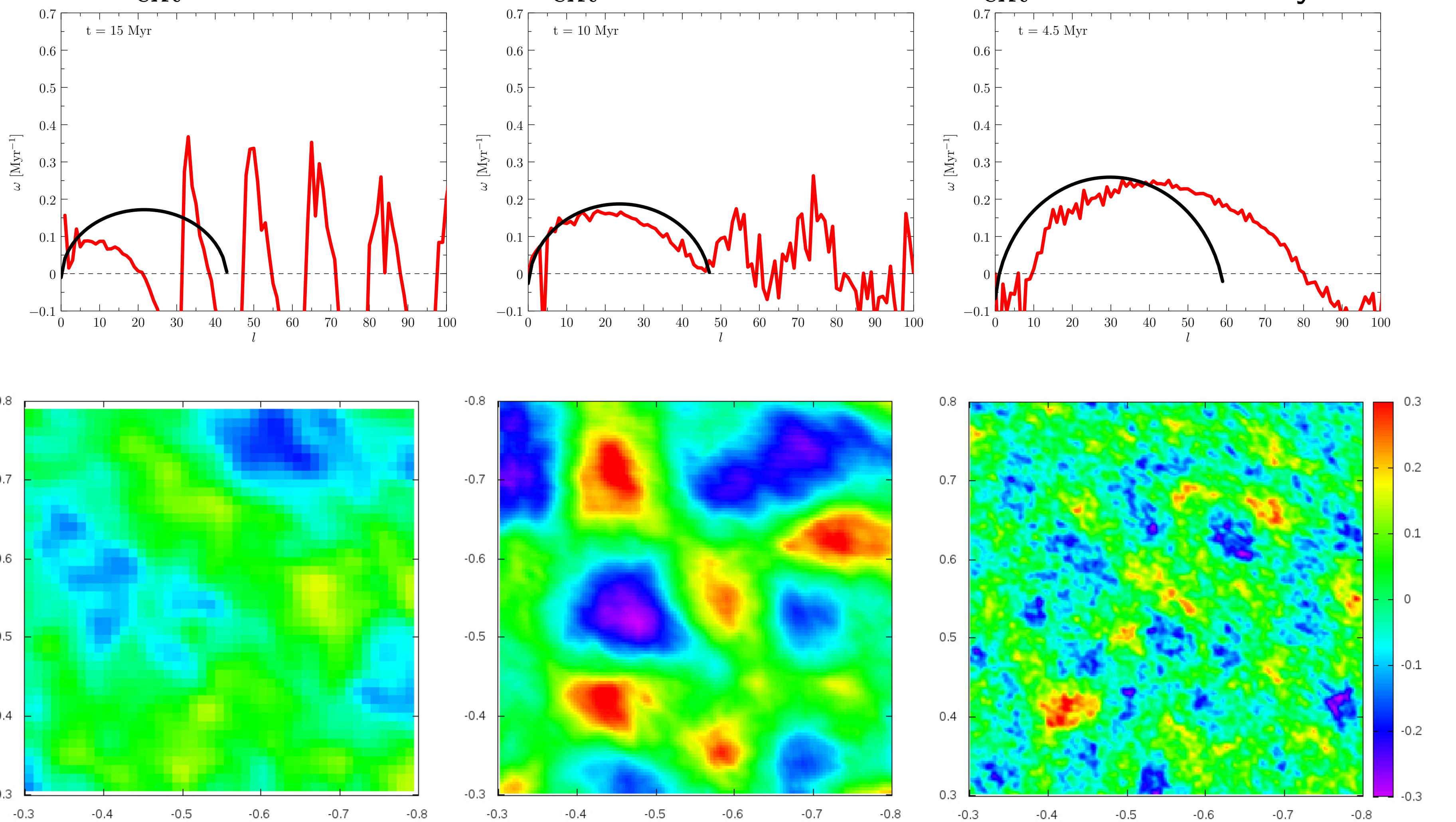


High pressure ambient medium

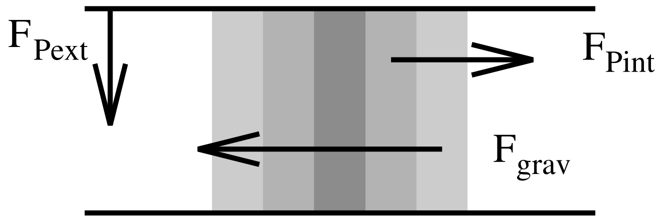


Dispersion relation depends on $P_{\rm ext}$ (and hence on the shell thickness)

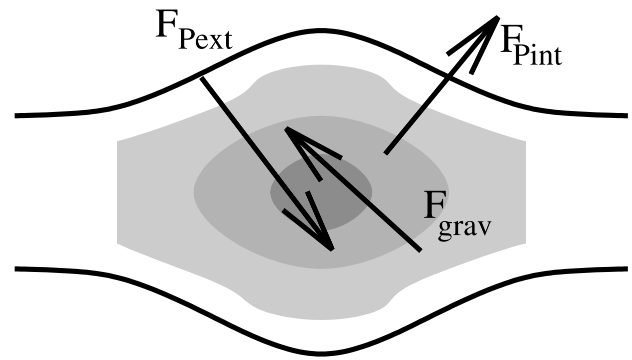
$P_{\rm ext} = 10^{-17}$ $\quad$ $P_{\rm ext} = 10^{-13}$ $\quad$ $P_{\rm ext} = 5 \times 10^{-13}$ dyne cm^{-2}



Dependence of fragment growth rate on $P_{\rm ext}$

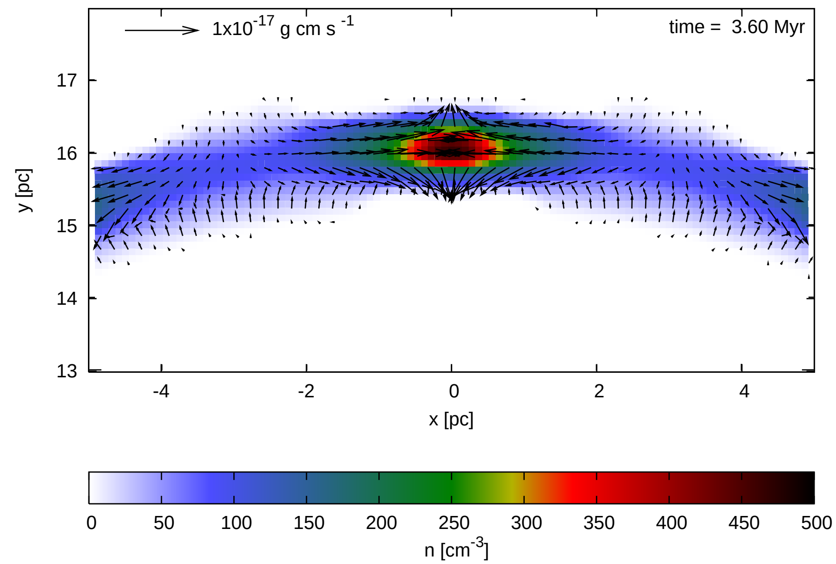


thin shell approx.



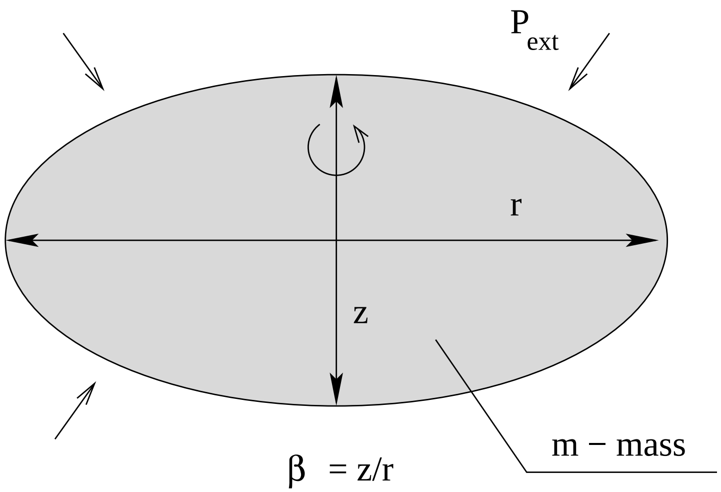
fragments in simulations

- the shell thickness varies across fragments
- external pres. force is not perpendicular to the shell surface



Uniform Oblate Spheroid

(Boyd & Whitworth, 2005)



Kinetic, gravitational and compressional energy:

$\mathcal{K} = \frac{m}{10}\left(2\dot{r}^2 + \dot{z}^2\right)$

$\mathcal{G} = -\frac{3\,G\,m^2}{5}\,\frac{\cos^{-1}(z/r)}{\left(r^2-z^2\right)^{1/2}}$

$\frac{d\mathcal{B}}{dV} = P_{\rm ext} - P_{\rm int} = P_{\rm ext} - \frac{m\,c_s^2}{V}$

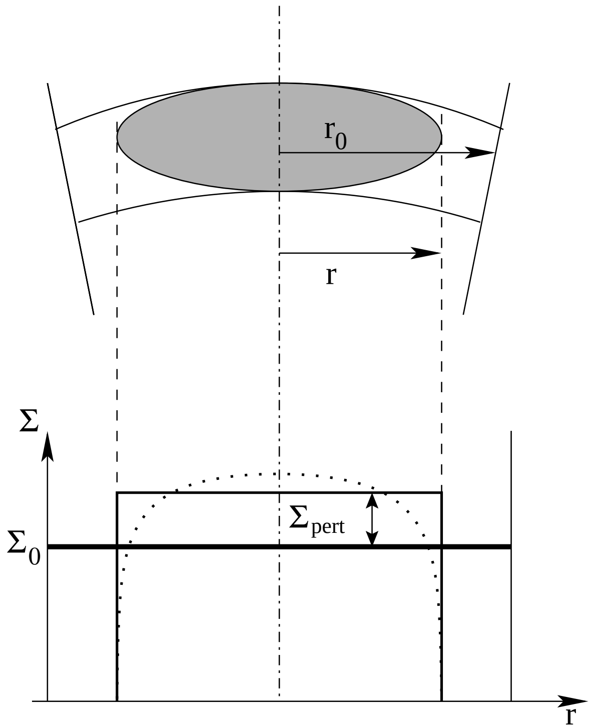
- energy conservation: $\mathcal{E} = \mathcal{K} + \mathcal{G} + \mathcal{B}$
- differentiate with resp. to time:

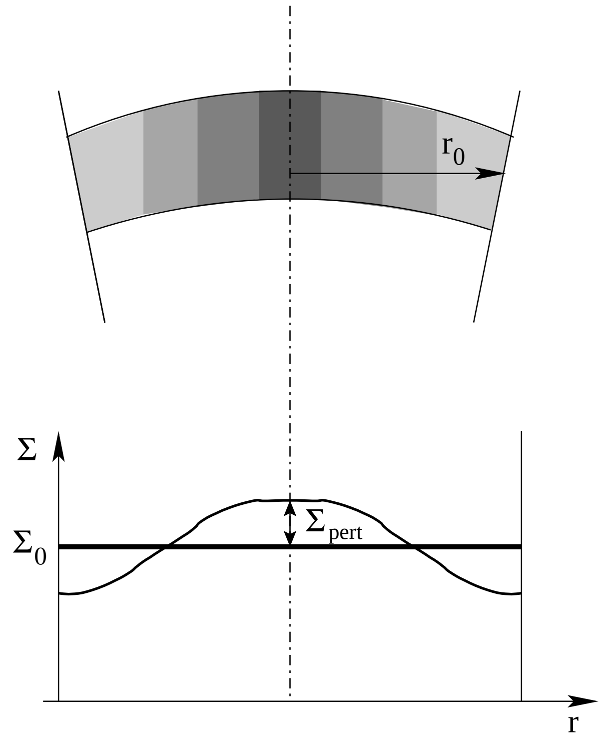
$$\left\{\frac{2\,m\,\ddot{r}}{5} + \frac{d\mathcal{G}}{dr} + \frac{d\mathcal{B}}{dr}\right\}\dot{r} + \left\{\frac{m\,\ddot{z}}{5} + \frac{d\mathcal{G}}{dz} + \frac{d\mathcal{B}}{dz}\right\}\dot{z} = 0\,.$$

- $\dot{r}$ and $\dot{z}$ independent:

$$\ddot{r} = -\frac{3Gm}{2r^2}\left[\frac{\cos^{-1}\beta}{(1-\beta^2)^{3/2}} - \frac{\beta}{1-\beta^2}\right] - \frac{20\pi P_{\rm ext} r z}{3m} + \frac{5c_s^2}{r}$$

UOS vs. thin shell





- z given by hydrostatic equilibrium
- non-linear eqs. of motion
- homogeneous ellipsoid

- no vertical structure (inf. thin shell)
- linearized hydrodyn. eqs.
- sinusoidal perturbations

Perturbation Growth Rate of the mode l

- wavenumber - size of fragment:

$$r_0 = \frac{\pi}{l} R, \qquad \dot{r}_0 = \frac{\pi}{l} V$$

- shell surface density - mass of fragment:

$$m = \pi r^2 \Sigma$$

- fragment collapses with const. acceleration $\ddot{r}$

$$r(t) = r_0 + \dot{r}_0 t + \frac{1}{2} \ddot{r} t^2$$

- shrinks by factor $(1 - \epsilon)$ during time t_ϵ: $r(t_\epsilon) = (1 - \epsilon) r_0$
- perturbation growth rate:

$$\omega_\epsilon = 1/t_\epsilon = -\frac{\dot{r}_0}{2 r_0 \epsilon} + \left(\frac{\dot{r}_0^2}{4 r_0^2 \epsilon^2} - \frac{\ddot{r}}{2 r_0 \epsilon} \right)^{1/2} \qquad (1)$$

- factor ϵ unknown; can be determined by comparison with ω_{thin}
- only range of unstable wavenumbers and relative growth rates

Dispersion relation of the thick shell

$$\omega_\epsilon = \underbrace{-\frac{V}{2R\epsilon} + \left\{\frac{V^2}{4R^2\epsilon^2}\right.}_{\text{stretching}} + \underbrace{\frac{3G\Sigma_0 l}{4R\epsilon}\left[\frac{\cos^{-1}\beta}{(1-\beta^2)^{3/2}} - \frac{\beta}{1-\beta^2}\right]}_{\text{gravity}}$$

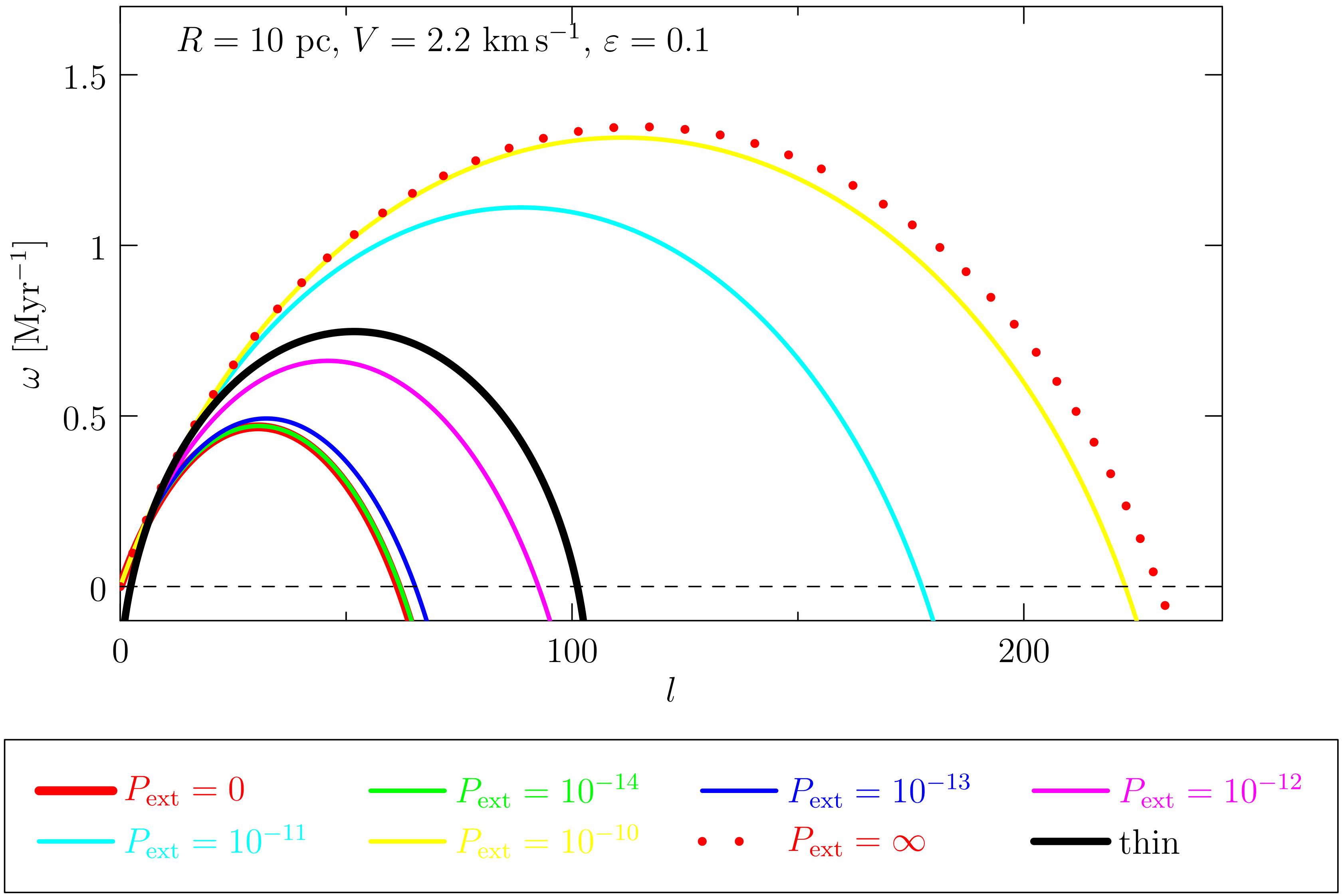
$$\left. + \underbrace{\frac{1 - P_{\text{ext}} c_s^2 l^2}{3\pi^2 R^2 \epsilon (2P_{\text{ext}} + \pi G \Sigma_0^2)}}_{\text{external pres.}} \underbrace{-\frac{5c_s^2 l^2}{2\pi^2 R^2 \epsilon}}_{\text{internal pres.}} \right\}^{1/2} .$$

Conf. to thin shell:

$$\omega_{\text{thin}} = \underbrace{-\frac{3V}{2R} + \left(\frac{V^2}{4R^2}\right.}_{\text{stretching}} + \underbrace{\frac{2\pi G \Sigma_0 l}{R}}_{\text{gravity}} \underbrace{-\frac{c_s^2 l^2}{R^2}}_{\text{internal pres.}} \left.\vphantom{\frac{V^2}{4R^2}}\right)^{1/2}$$

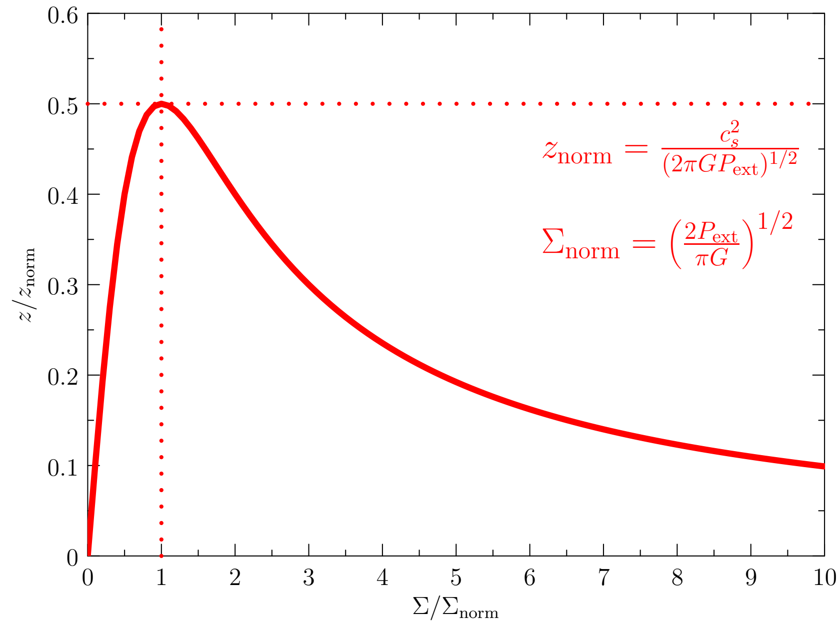
- geometry factor at gravity term
- external pressure term
- dependence on ϵ - shrink fraction

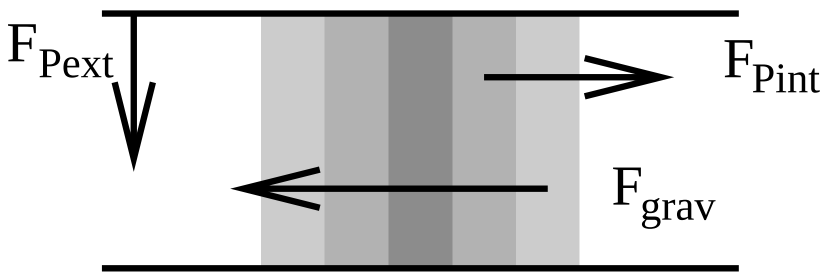
Pressure assisted gravitational instability

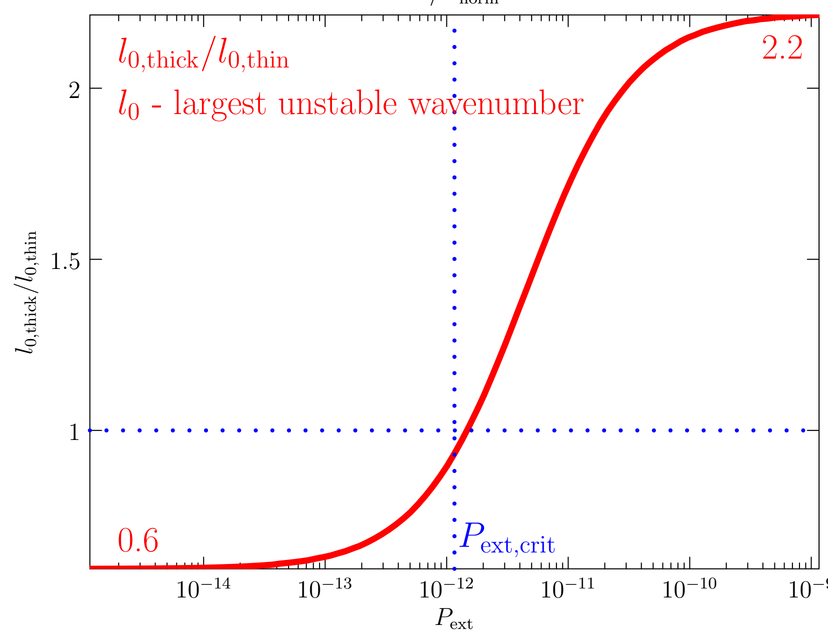


Critical external pressure: $\omega_\epsilon \sim \omega_{\rm thin}$

- $\omega_\epsilon \sim \omega_{\rm thin}$ for $P_{\rm ext,crit}$ for which $\frac{dz}{d\Sigma} = 0$
  (the shell thickness does not depend on the surface density)

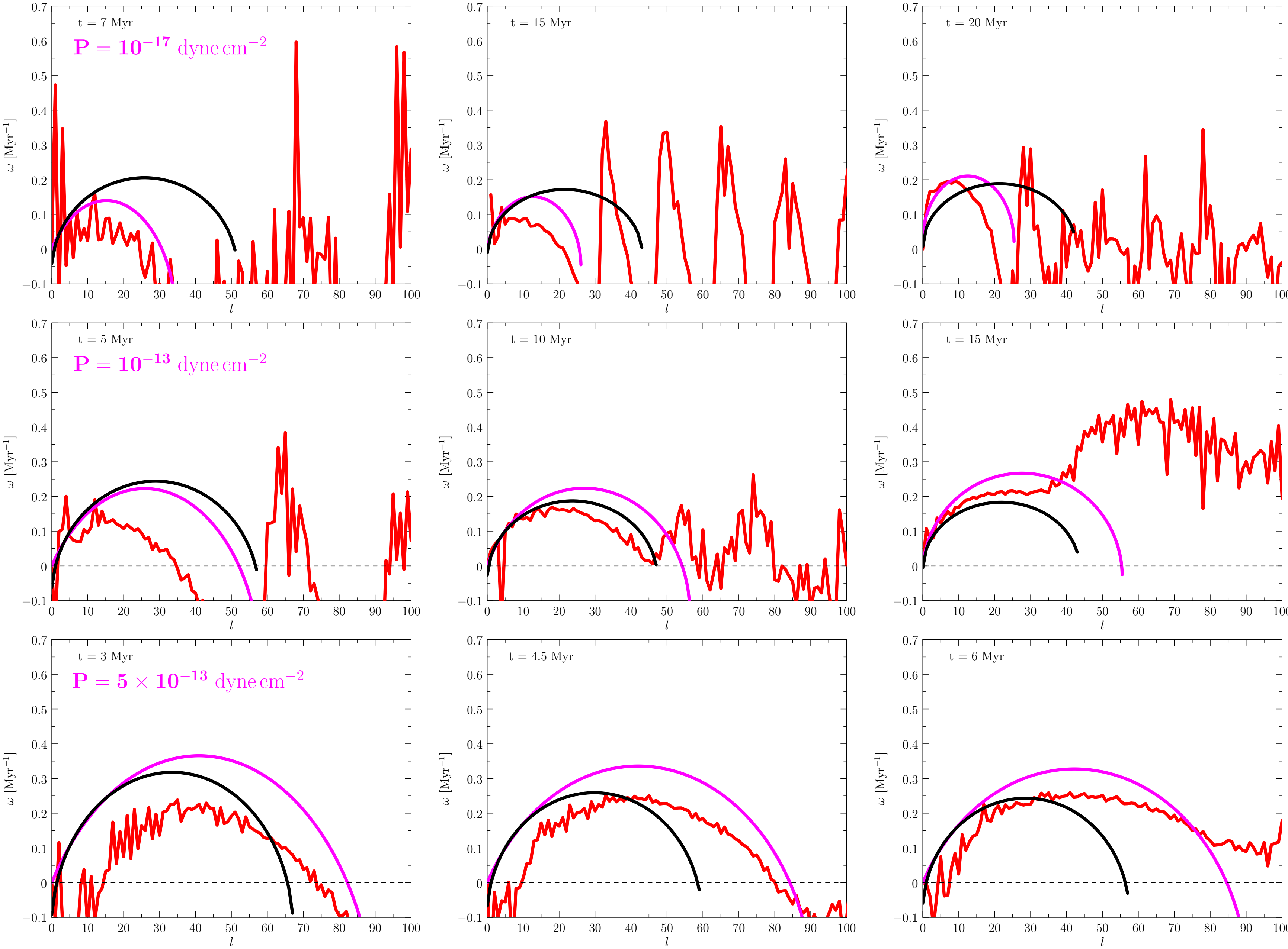






thin shell approx.

PAGI vs. simulations



Discussion

- PAGI gives systematically larger range of unstable wavelengths than simulations; most probably due to approximation of the UOS evolution by constant acceleration
- ω only amplifies initial spectrum (given probably mainly by the turbulence)
- we assume shell confined by thermal pressure from both sides, but instability can be different if the ram pressure confines the shell from outside (accreting shell)
- Vishniac and RT instabilities may have an impact
- astrophysical consequences: top-heavy IMF (deficit of low mass fragments) in low pres environments
  observational proposal: we plan to use APEX to determine fragment mass function of the Carina Flare supershell (450 pc above the Galactic plane)

Conclusions

- excellent agreement between AMR and SPH, but disagreement with the thin shell approximation
- new instability (PAGI) and dispersion relation (fragment growth rate) for the thick shell embedded in the medium with non-zero pressure
- the PAGI dispersion relation depends on the external pressure, predicts range of unstable wavenumbers different than the one given by the thin shell approximation by factor of 0.6 and 2.2 for $P_{\rm ext} = 0$ and ∞, respectively
- the PAGI dispersion relation is similar to the thin shell one for the pressure for which the shell thickness locally does not depend on its surface density (maximum shell thickness)

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