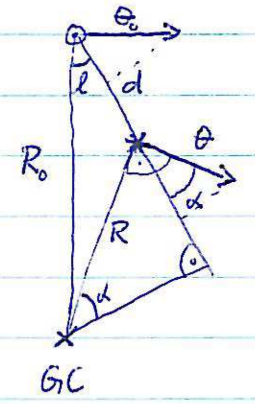
poloha hvězdy : $l, b, d, \mu_l \cos b, \mu_b, V_R$

Radial :

$$V_R = \theta \cos\alpha - \theta_0 \sin l$$



$$\frac{\sin l}{R} = \frac{\sin(90+\alpha)}{R_0} = \frac{\cos\alpha}{R_0}$$

$$V_R = \theta \frac{R_0}{R} \sin l - \theta_0 \sin l$$

$$\omega \equiv \omega(R)$$
$$\omega_0 \equiv \omega(R_0)$$

$$\underline{V_R = (\omega - \omega_0) R_0 \sin l}$$

Tangential :

$$V_T = \theta \sin\alpha - \theta_0 \cos l$$

$$R \sin\alpha = R_0 \cos l - d$$

$$V_T = \frac{\theta}{R}(R_0 \cos l - d) - \theta_0 \cos l$$

$$\underline{V_T = (\omega - \omega_0) R_0 \cos l - \omega d}$$

Pro blízké hvězdy : $d \ll R_0$ $\quad (d \lessapprox 1\,\text{kpc})$

$$\omega(R) \doteq \omega(R_0) + \left(\frac{d\omega}{dR}\right)_0 (R - R_0) + \dots$$

$$\frac{d\omega}{dR} = \frac{d}{dR}\left(\frac{\theta}{R}\right) = \frac{1}{R}\frac{d\theta}{dR} - \frac{\theta}{R^2}$$

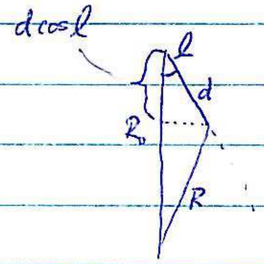
$$\left(\frac{d\omega}{dR}\right)_0 = \frac{1}{R_0}\left(\frac{d\theta}{dR}\right)_0 - \frac{\theta_0}{R_0^2}$$

Radial :

$$V_R = (\omega - \omega_0) R_0 \sin l = \left(\frac{d\omega}{dR}\right)_0 (R - R_0) R_0 \sin l$$

$$V_R = \left[\left(\frac{d\theta}{dR}\right)_0 - \frac{\theta_0}{R_0}\right](R - R_0) \sin l$$

$$R_0 - R \doteq d \cos l$$



$$V_R = \underbrace{\frac{1}{2}\left[-\left(\frac{d\theta}{dR}\right)_0 + \frac{\theta_0}{R_0}\right]}_{A} d \sin 2l$$

$$\boxed{V_R = A d \sin(2l)}$$

<u>Tangential:</u>

$$V_T = (\omega - \omega_0) R_0 \cos l - \omega d \qquad \Big| \; \omega d = \omega_0 d + \cancel{(\omega - \omega_0) d}$$

$$V_T = \left[\left(\frac{d\theta}{dR}\right)_0 - \frac{\theta_0}{R_0}\right](R - R_0) \cos l - \omega_0 d$$

$$V_T = \left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_0\right] d \cos^2 l - \omega_0 d$$

$$\cos^2 l = \frac{1}{2}(1 + \cos 2l)$$

$$V_T = \frac{1}{2}\left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_0\right] d - \frac{\theta_0}{R_0} d + \underbrace{\frac{1}{2}\left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_0\right]}_{A} d \cos 2l$$

$$V_T = \frac{1}{2}\left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_0\right] d \cos 2l \underbrace{- \frac{1}{2}\left[\frac{\theta_0}{R_0} + \left(\frac{d\theta}{dR}\right)_0\right]}_{B} d$$

$$\boxed{V_T = A d \cos 2l + B d} \qquad \boxed{\mu_l = \frac{V_T}{4{,}74 d} = \frac{A \cos 2l + B}{4{,}74}}$$

$$A = \frac{1}{2}\left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_0\right] \simeq 13 \text{ km} \cdot \text{s}^{-1} \text{ kpc}^{-1} \qquad (14{,}82 \pm 0{,}84)$$

$$B = -\frac{1}{2}\left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_0\right] = -13 \text{ km} \cdot \text{s}^{-1} \text{ kpc}^{-1} \qquad (-12{,}37 \pm 0{,}64)$$

$$A + B = -\left(\frac{d\theta}{dR}\right)_0 \sim 0 \text{ km s}^{-1} \text{ kpc}^{-1}$$

$$A - B = \frac{\theta_0}{R_0} \equiv \omega_0 \sim 26 \text{ km} \cdot \text{s}^{-1} \text{ kpc}^{-1}$$

$$R_0 = 8{,}5 \text{ kpc}$$

$$T_0 = \frac{2\pi}{\omega_0} = 250 \text{ Myr}$$

Rotation curve depending on F_R

gravitational force: $F_R = \frac{\theta^2}{R}$ - balanced by centrifugal f.

$$\frac{dF_R}{dR} = -\frac{\theta^2}{R^2} + 2\frac{\theta}{R}\frac{d\theta}{dR}$$

$$\Rightarrow \frac{d\theta}{dR} = \frac{\theta}{2R} + \frac{dF_R}{dR}\frac{R}{2\theta}$$

$$A = \frac{1}{2}\left[\frac{\theta}{R} - \frac{d\theta}{dR}\right] = \frac{1}{2}\left[\frac{\theta}{R} - \frac{\theta}{2R} - \frac{dF_R}{dR}\frac{R}{2\theta}\right] =$$

$$= \frac{1}{4}\frac{\theta}{R}\left[1 - \frac{R}{F_R}\frac{dF_R}{dR}\right]$$

$$B = -\frac{1}{2}\left[\frac{\theta}{R} + \frac{d\theta}{dR}\right] = -\frac{1}{2}\left[\frac{\theta}{R} + \frac{\theta}{2R} + \frac{dF_R}{dR}\frac{R}{2\theta}\right] =$$

$$= -\frac{1}{4}\frac{\theta}{R}\left[3 + \frac{R}{F_R}\frac{dF_R}{dR}\right]$$

Examples:

- $F_R = \text{const}\cdot R$: $\frac{dF_R}{dR} = \text{const}$ (homogeneous sphere)

$$A = 0, \quad B = -\frac{\theta}{R}$$

$$\omega = -B = \text{const}$$

$$\frac{d\omega}{dR} = \frac{d}{dR}\left(\frac{\theta}{R}\right) = -\frac{\theta}{R^2} + \frac{1}{R}\frac{d\theta}{dR} = 0 \Rightarrow \text{solid body rotation.}$$

$$A = \frac{1}{2}\left[\frac{\theta}{R} - \frac{d\theta}{dR}\right] = 0$$

$F_R = \frac{\text{const}}{R^2}$: $\frac{dF_R}{dR} = -\frac{2\,\text{const}}{R^3}$

$$A = \frac{3}{4}\frac{\theta}{R}, \quad B = -\frac{1}{4}\frac{\theta}{R}$$

$F_R = \frac{\text{const}}{R}$: $\frac{dF_R}{dR} = -\frac{\text{const}}{R^2}$ - close to galactic

$$A = \frac{1}{2}\frac{\theta}{R}, \quad B = -\frac{1}{2}\frac{\theta}{R} \quad \Rightarrow \Phi(R) \sim \ln(R)$$