Záření ISM

HI - čára 21cm

Plank's law: $I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \equiv B_\nu$

Brightness: $I_\nu d\nu$ ... amount of energy per unit surface per unit time per unit solid angle emitted in the fq. range $(\nu, \nu + d\nu)$

Flux: $S_\nu = \int I_\nu(\vartheta, \varphi) \cos\vartheta \, d\Omega$

unit: $Jy = 10^{-26} \frac{W}{m^2 \cdot Hz} = 10^{-23} \frac{erg}{s \cdot cm^2 \cdot Hz}$

(Karl Guthe Jánský; 1931 discovered radio waves coming from MW)

Energy density: $u_\nu(\Omega) = \frac{1}{c} I_\nu$ (per solid angle)

$\boxed{u_\nu = \frac{4\pi}{c} I_\nu}$ $\quad u_\nu = \cancel{\int_{4\pi} u_\nu(\Omega) d\Omega} = \frac{1}{c} \int I_\nu d\Omega$ (total)

? (may be just $u_\nu = \frac{4\pi}{c} I_\nu$) - en. dens per volume

or power integrated over all solid angles:

$u = \int I_\nu \cos\vartheta \, d\Omega = \int I_\nu \cos^2\vartheta \, d\vartheta \, d\varphi = \pi I_\nu$

Rayleigh - Jeans law: for $h\nu \ll kT$

$\exp\left(\frac{h\nu}{kT}\right) = 1 + \frac{h\nu}{kT} \ldots \Rightarrow B_\nu = \frac{2\nu^2}{c^2} kT$

Brightness temperature: T_B ... temperature assigned to an object by Plank's (R-J) law according to its brightness

Absorption: $$\frac{dI_\nu}{dl} = -\varkappa_\nu I_\nu$$

$\varkappa_\nu$ - absorption koefficient, opacity

$d\tau_\nu = \varkappa_\nu \, dl$ - optical depth

$$\tau_\nu = \int_0^l \varkappa_\nu \, dl$$

solution: $I_\nu = I_{\nu,0} \exp(-\tau_\nu)$

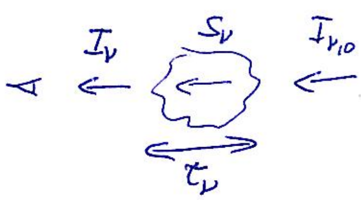
Radiation transport equation:

+ emission: $$\boxed{\frac{dI_\nu}{dl} = -\varkappa_\nu I_\nu + \varepsilon_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{\varepsilon_\nu}{\varkappa_\nu} = -I_\nu + S_\nu$$ ← source function

Formal solution:

$$I_\nu = I_{\nu,0} \exp(-\tau_\nu) + \int_0^{\tau_\nu} S_\nu \exp(-\tau_\nu') \, d\tau_\nu'$$



assuming: $S_\nu \neq S_\nu(\tau_\nu)$, $I_{\nu,0} = \emptyset$

$$I_\nu = S_\nu \int_0^{\tau_\nu} \exp(-\tau_\nu') \, d\tau_\nu' = S_\nu [1 - \exp(-\tau_\nu)]$$

a) optically thick: $\tau_\nu \gg 1$: $1 - \exp(-\tau_\nu) = 1$

$$I_\nu = S_\nu$$

b) optically thin: $\tau_\nu \ll 1$: $1 - \exp(-\tau_\nu) = \tau_\nu$

$$I_\nu = S_\nu \tau_\nu$$

Local Thermodynamic Equilibrium

$S_\nu = B_\nu(T_k)$; T_k ... kinetic temperature

from R-J. law : $S_\nu = B_\nu \sim T_k$

$\Rightarrow$ we can define brightness temperature : $T_B \equiv \frac{c^2}{2\nu^2 k} I_\nu$

$$T_B = T_k \left[1 - \exp(-\tau_\nu) \right]$$

a) optically thick : $T_B = T_k$

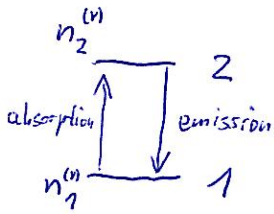
b) optically thin : $T_B = T_k \tau_\nu$

Antenna temperature : $T_A = \alpha T_B$

beam dilution (fraction of the beam filled by the object)

Two energy-level model

$$\frac{dI_\nu}{dl} = \frac{h\nu}{4\pi}\left[n_2^{(\nu)} A_{21} - \left(n_1^{(\nu)} B_{12} - n_2^{(\nu)} B_{21} \right) \frac{4\pi I_\nu}{c} \right]$$



$n_{1,2}^{(\nu)}$ ... number of atoms able to emit (2) or absorp (1) photon of the fq. ν

Einstein coefficiens : A_{21} - spontaneous ~~em~~emission

B_{21} - stimulated emission

B_{12} - absorption

$$\varkappa_\nu \equiv \frac{h\nu}{c}\left(n_1^{(\nu)} B_{12} - n_2^{(\nu)} B_{21} \right)$$

$$S_\nu \equiv \frac{c}{4\pi} \frac{n_2^{(\nu)} A_{21}}{n_1^{(\nu)} B_{12} - n_2^{(\nu)} B_{21}}$$

in LTE:

$$\frac{n_2^{(\nu)}}{n_1^{(\nu)}} = \frac{g_2}{g_1} \exp\left(-\frac{h\nu}{kT}\right) \doteq \frac{g_2}{g_1}$$

$h\nu \ll kT$

g_i ... statistical weights, degeneracy of the level

Einstein's relations:

$$A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21}$$

$$g_1 B_{12} = g_2 B_{21}$$

Optical depth:

$$\tau_\nu = \int_0^l \varkappa_\nu \, dl' = \frac{h\nu}{c}\int_0^l \left(n_1^{(\nu)} B_{12} - n_2^{(\nu)} B_{21}\right) dl'$$

$$n_1 B_{12} - n_2 B_{21} = \frac{c}{4\pi} \frac{n_2 A_{21}}{S_\nu} = \frac{c}{4\pi} \frac{n_2 A_{21} c^2}{2kT\nu^2} = \frac{\cancel{c}}{\cancel{4\pi}} \frac{n_2 \cancel{c^2}}{2kT\cancel{\nu^2}} \frac{\cancel{8\pi} h\nu^{\cancel{3}}}{\cancel{c^3}} B_{21}$$

$$= \frac{h\nu}{kT} n_2 B_{21} = \frac{h\nu}{kT} n_1 \frac{\cancel{g_2}}{\cancel{g_1}} B_{12} \frac{\cancel{g_1}}{\cancel{g_2}}$$

$$\tau_\nu = \frac{h^2\nu^2}{ckT} B_{12} \int_0^l n_1^{(\nu)} \, dl' = \frac{B_{12} h^2 \nu^2}{ck} \frac{N_1^{(\nu)}}{T} \quad ; \quad N_1^{(\nu)} = \int_0^l n_1^{(\nu)} \, dl'$$

Column density - Brightness temperature relation

$$\tau_\nu = \frac{B_{12} h^2 \nu^2}{ck} \frac{N_1^{(\nu)}}{T}$$

$$N_1^{(\nu)} = \frac{ck}{B_{12} h^2 \nu^2} \, T \tau_\nu$$ — brightness temp. T_B

$$N_1 = \int_0^\infty N_1^{(\nu)} d\nu$$

but let's do it in velocities:

$$\frac{v}{c} = 1 - \frac{\nu}{\nu_0} \quad \Rightarrow \quad \nu = \nu_0 \left(1 - \frac{v}{c}\right), \quad d\nu = -\frac{\nu_0}{c} dv$$

$$N_1^{(\nu)} d\nu = N_1^{(v)} dv$$

$$\Rightarrow N_1^{(v)} = \frac{ck \, T_B(v)}{B_{12} h^2 \nu_0^2 \underbrace{\cancel{\left(1 - \frac{v}{c}\right)}}_{\text{close to } 1}} \frac{d\nu}{dv} = - \frac{ck \, T_B(v) \cancel{\nu_0}}{B_{12} h^2 \nu_0^{\cancel{2}} \cancel{c}}$$

$$N_1 = - \frac{k}{B_{12} h^2 \nu_0} \int_\infty^{-\infty} T_B(v) \, dv = \frac{k}{B_{12} h^2 \nu_0} \int_{-\infty}^{\infty} T_B(v) \, dv$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} = 3, \quad N_H = N_1 + N_2 = 4 N_1$$

$$\boxed{N_H = \frac{4k}{B_{12} h^2 \nu_0} \int_{-\infty}^{\infty} T_B(v) \, dv = 1{,}82 \cdot 10^{18} \int_{-\infty}^{\infty} \frac{T_B(v)}{K} dv \ \mathrm{cm}^{-2}}$$