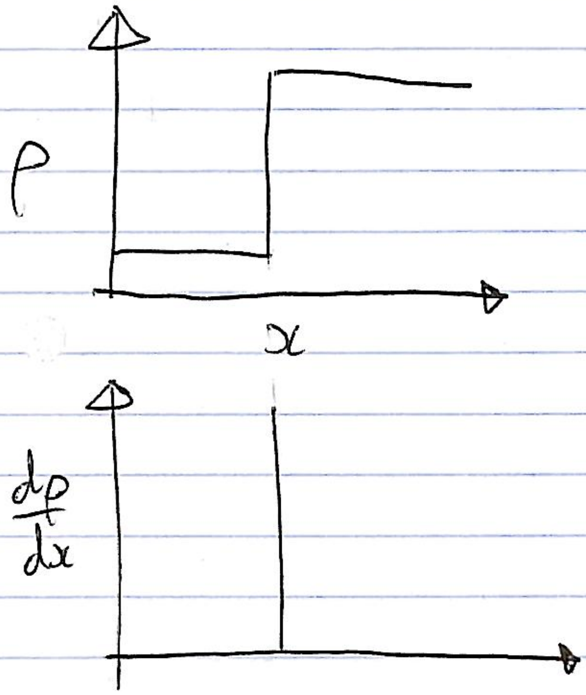
# SHOCKS

What is a shock?

Very sharp change in fluid velocity
pressure
density
temperature (sometimes)

over a very short distance



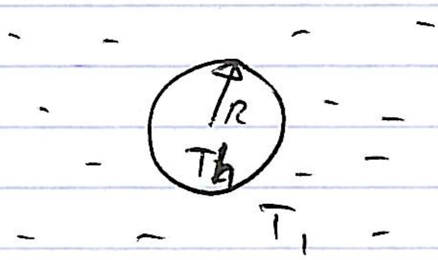

$\cancel{\frac{d}{dt}}$ $\frac{dv}{dx}, \frac{dP}{dx}, \frac{d\rho}{dx}, \frac{dT}{dx}$ very large

Occurs when something moves through the fluid faster than the speed of sound.

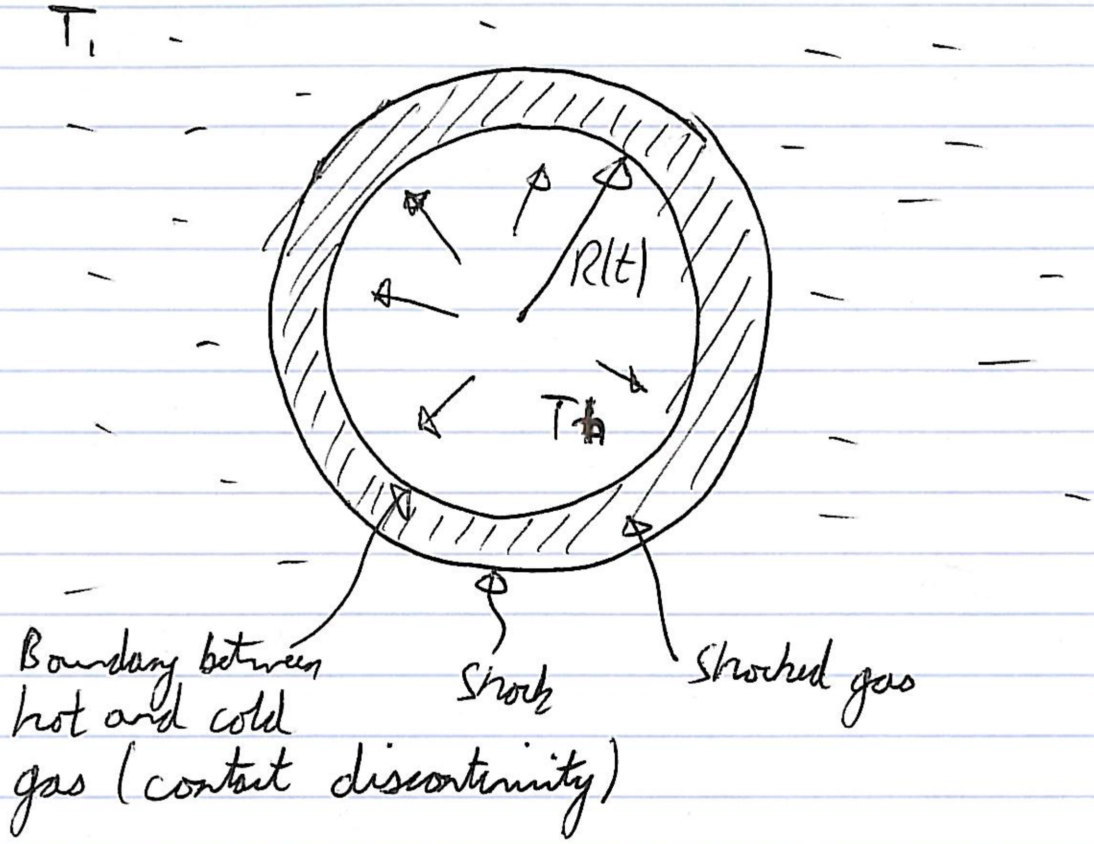
Fluid cannot move out of the way.

Instead collects in front of moving object.

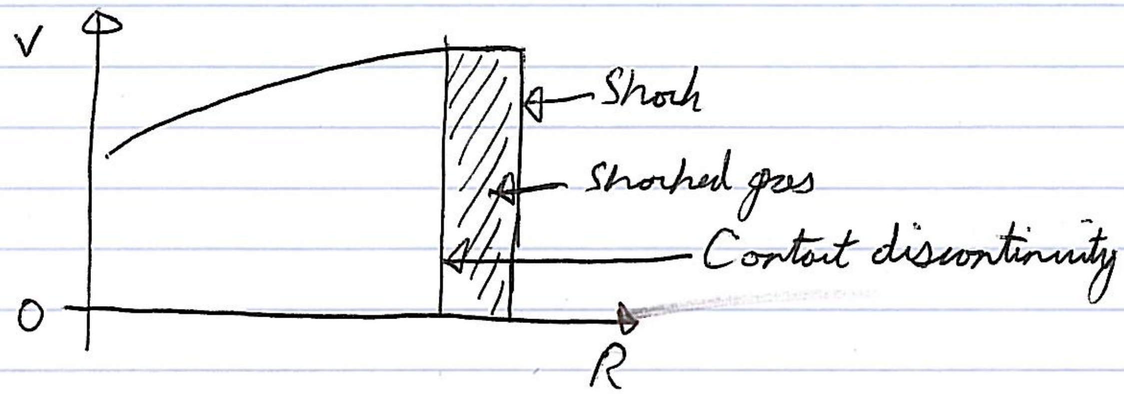
Suppose we have a uniform gas at temperature $T_1$, and we suddenly heat a sphere of radius $R$ to $T_h \gg T_1$ (explosion)



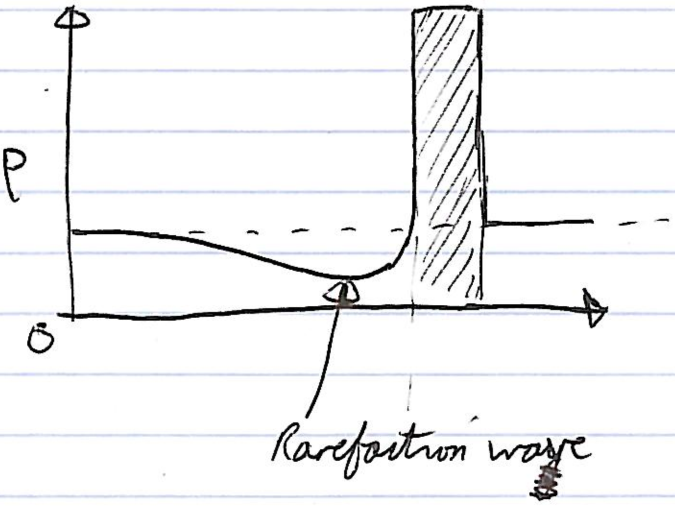

Hot gas expands at speed of sound in hot gas — $c_s \sim T^{1/2}$
$T_h \gg T_1 \Rightarrow c_{s,2} > c_{s,1} \Rightarrow$ bubble expands
supersonically



$T_1$
$R(t)$
$T_h$
Boundary between hot and cold gas (contact discontinuity)
Shock
Shocked gas

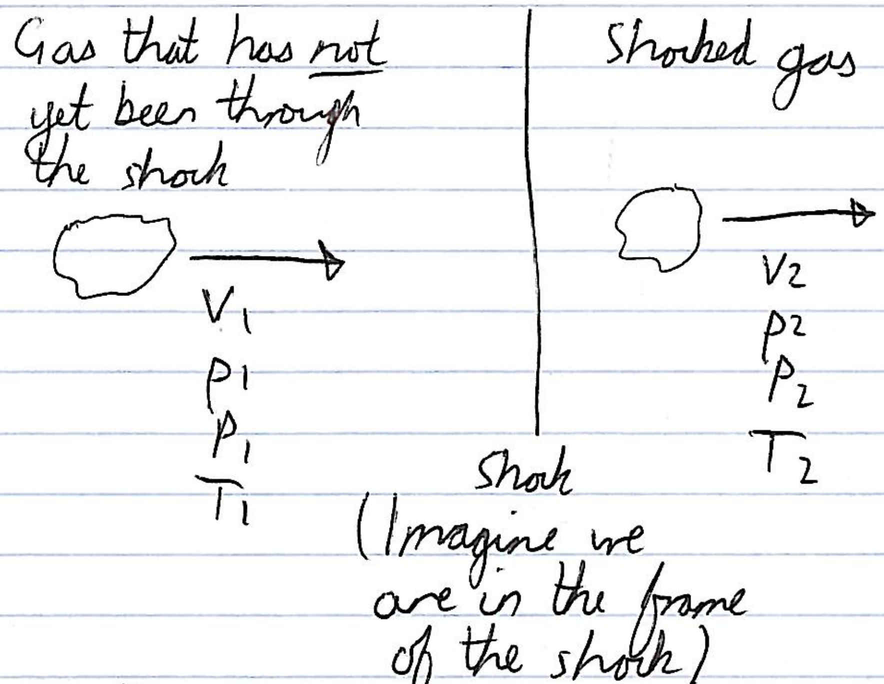



V
0
R
Shock
Shocked gas
Contact discontinuity

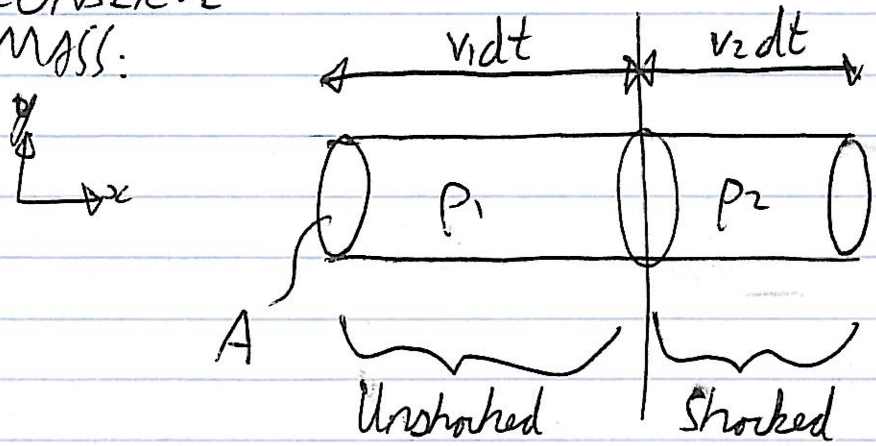



P
0
Rarefaction wave

## What happens to the gas at the shock?




(i) CONSERVE MASS:




$$\rho_1 v_1 dt A = \rho_2 v_2 dt A$$

$$\Rightarrow \boxed{\rho_1 v_1 = \rho_2 v_2}$$

## (ii) CONSERVE MOMENTUM:

Force acting in $-x$ direction

$$\underbrace{(\rho_2 v_2\, dt A) v_2 - (\rho_1 v_1\, dt A) v_1}_{\text{Change in momentum caused by the shock}} = \underbrace{-(P_2 - P_1)\, dt A}_{\text{Impulse } (F dt)}$$

$$\Rightarrow \boxed{\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2}$$

## (iii) CONSERVE ENERGY: (Assume shock is adiabatic)

Internal energy per unit mass

$$\underbrace{\frac{1}{2}(\rho_1 v_1\, dt A) v_1^2}_{\text{Kinetic energy}} + \underbrace{(\rho_1 v_1\, dt A)\, \varepsilon_1}_{\text{Internal energy}} + \underbrace{(v_1\, dt A) P_1}_{\text{Work done on/by gas}}$$

$P_1 \Delta V_1 - P_2 \Delta V_2$

$$= \frac{1}{2}(\rho_2 v_2\, dt A) v_2^2 + (\rho_2 v_2\, dt A)\, \varepsilon_2 + (v_2\, dt A) P_2$$

$$\Rightarrow \boxed{\frac{1}{2} v_1^2 + \varepsilon_1 + \frac{P_1}{\rho_1} = \frac{1}{2} v_2^2 + \varepsilon_2 + \frac{P_2}{\rho_2}}$$

Not nice...

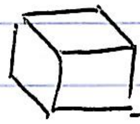
Internal energy per atom

$$E = F \frac{k_B T}{2}$$

$F$ = no. of atomic/molecular degrees of freedom
= 3 for atomic
5 for cold diatomic
... etc

$c_v$ = heat capacity at constant volume

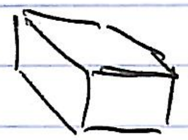
$$= \frac{E}{T} = \frac{F k_B}{2}$$



$$\Delta U = N F \frac{k_B}{2} \Delta T$$

$c_p$ = heat capacity at constant pressure

$$c_p = c_v + k_B$$



$$\Delta U = N F \frac{k_B}{2} \Delta T + P \Delta V$$

$$= N F \frac{k_B}{2} \Delta T + N k_B \Delta T$$

Define $\gamma = \frac{c_p}{c_v}$

$$\Rightarrow c_v(\gamma - 1) = k_B$$

$$\Rightarrow F \frac{k_B}{2}(\gamma - 1) = k_B$$

$$F = \frac{2}{(\gamma - 1)}$$

$$\Rightarrow \underset{\text{(per atom)}}{E} = \frac{2}{(\gamma-1)} \frac{k_B T}{2} = \frac{k_B T}{(\gamma - 1)}$$

$$P = n k_B T = \frac{\rho k T}{\bar{m}}$$

$\bar{m}$ = mean atomic / molecular mass

$\Rightarrow \frac{E}{\bar{m}}$ = energy per unit mass = $\varepsilon$

$$= \frac{k_B T}{\bar{m}(\gamma - 1)} = \frac{1}{(\gamma - 1)} \frac{P}{\rho}$$

$$\Rightarrow \frac{1}{2}v_1^2 + \frac{1}{(\gamma-1)}\frac{P_1}{\rho_1} + \frac{P_1}{\rho_1} = \frac{1}{2}v_2^2 + \frac{1}{(\gamma-1)}\frac{P_2}{\rho_2} + \frac{P_2}{\rho_2}$$

$$\Rightarrow \boxed{\frac{1}{2}v_1^2 + \left(\frac{\gamma}{\gamma-1}\right)\frac{P_1}{\rho_1} = \frac{1}{2}v_2^2 + \left(\frac{\gamma}{\gamma-1}\right)\frac{P_2}{\rho_2}}$$

So, if $\gamma = 5/3$ (diatomic gas, eg $H_2$)

$\rho_2 = 4\rho_1$ – gas becomes 4x denser

$v_2 = \frac{v_1}{4}$ – gas slows by factor of 4.

$\frac{P_2}{\rho_2} = \frac{kT_2}{\bar{m}} = \frac{3}{16}v_1^2$

Ram pressure $\rightarrow$ thermal pressure
Kinetic energy $\rightarrow$ Heat

If $v_1 = 1000\ kms^{-1}$ (Stellar wind)

$T_2 \simeq 2\times10^7\ K$ – X-rays

But what if the hot gas cools very fast?

- Shock isothermal instead of adiabatic
- Equation (iii) does not apply - shock is <u>not</u> conserving energy.

Then we just have

$$(i) \quad \rho_1 v_1 = \rho_2 v_2$$

$$(ii) \quad \rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2$$

and we assume $T_2 = T_1$ - shock does not change the gas temperature

$\Rightarrow$ speed of sound on both sides of the shock is the same $= c_s = \sqrt{\frac{P}{\rho}}$

$$\Rightarrow P_1 = \rho_1 c_s^2$$

$$P_2 = \rho_2 c_s^2$$

$$\Rightarrow \rho_1 (v_1^2 + c_s^2) = \rho_2 (v_2^2 + c_s^2)$$

$$\Rightarrow v_2 v_1 + \frac{v_2}{v_1} c_s^2 = v_2^2 + c_s^2$$

$$v_2 v_1^2 - v_1 v_2^2 = (v_1 - v_2) c_s^2$$

$$\Rightarrow v_2 v_1 = c_s^2$$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \left(\frac{v_1}{c_s}\right)^2 = \mathcal{M}^2$$

Mach Number

$\Rightarrow$ For $v_1 = 1000\ \mathrm{km\,s^{-1}}$

$c_s \approx 0.2\ \mathrm{km\,s^{-1}}$ in 10K H

$\mathcal{M} = 5000$

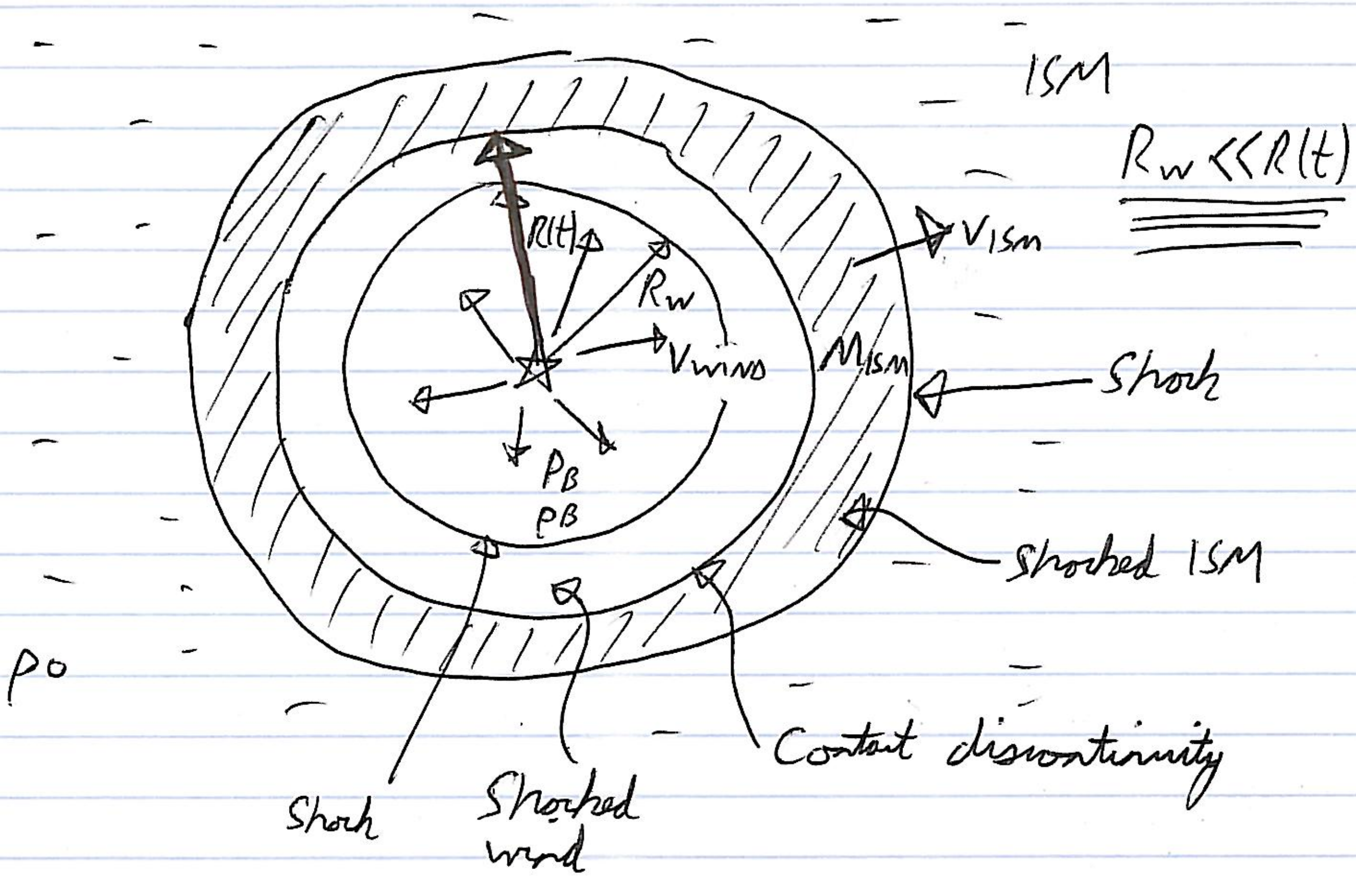
$\Rightarrow$ Very large density contrast

$v_2$ very small

$\Rightarrow$ A thin, dense layer collects behind the shock.

But what is the speed of the shock?

Consider a wind bubble (two shocks...)




Total energy injected by wind $= L_w t$
$L_w =$ wind 'luminosity' $= \frac{1}{2} \dot{M} V_{WIND}^2$

$t =$ time wind acting for

$\dot{M} =$ rate at which star is losing mass

Internal energy of wind bubble =

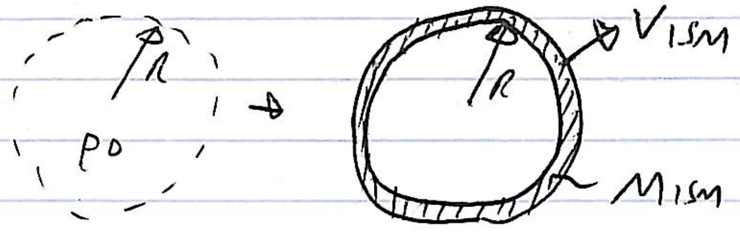
$$\underbrace{\frac{3}{2}\frac{kT}{\bar{m}}}_{\text{Internal energy per unit mass}} \times \underbrace{\frac{4}{3}\pi R^3 \rho_B}_{\text{Mass of bubble}} = \frac{3}{2}\frac{P_B}{\rho_B} \cdot \frac{4}{3}\pi R^3 \rho_B = 2\pi R^3 P_B = L_w t$$

$$\Rightarrow P_B = \frac{L_w t}{2\pi R^3}$$

Pressure causes the bubble to expand, driving the shell of shocked ISM:

$$\frac{d}{dt}\left(M_{ISM} V_{ISM}\right) = 4\pi R^2 P_B$$

$$M_{ISM} = \frac{4\pi R^3}{3}\rho_0$$




$$V_{ISM} = \frac{dR}{dt}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{4\pi R^3}{3}\rho_0 \frac{dR}{dt}\right) = 4\pi R^2 \frac{L_w t}{2\pi R^3}$$

$$\frac{R}{t}\frac{d}{dt}\left(R^3 \frac{dR}{dt}\right) = \frac{3 L_w}{2\pi \rho_0} \quad \ldots \quad ?$$

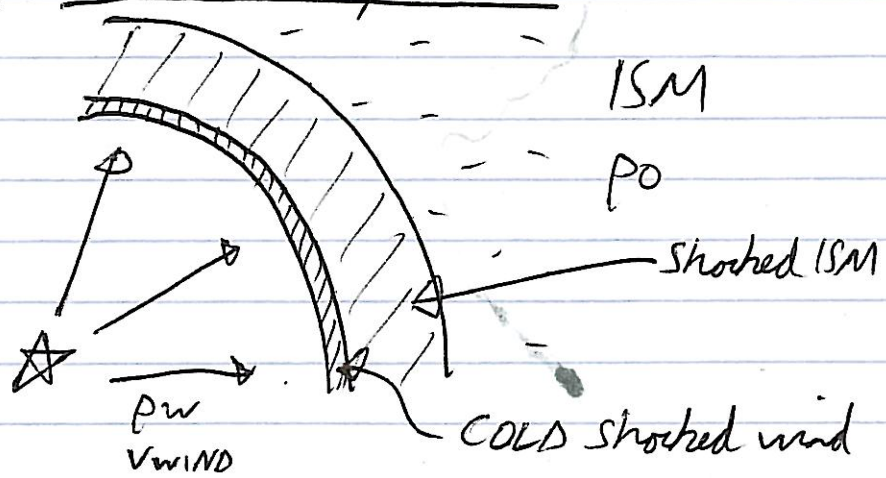
Try $R(t) = A t^n$

$$\Rightarrow R(t) = \left(\frac{25 L_w}{14\pi \rho_0}\right)^{1/5} t^{3/5}$$

– assumed all energy injected by the wind stays in the bubble.

what if the shocked wind cools?

Then $P_B$ is very small and the bubble is driven by <u>wind ram pressure</u>




Ram pressure $= \rho_w v_{WIND}^2$

$$\Rightarrow \frac{d}{dt}\left(\frac{4\pi R^3}{3}\rho_0 \frac{dR}{dt}\right) = 4\pi R^2 \rho_w v_{WIND}^2$$

$\rho_w = \rho_w(R)$ — By conservation of mass in the wind

$$4\pi R^2 \rho_w v_{WIND} = \dot{M}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{4\pi R^3}{3}\rho_0 \frac{dR}{dt}\right) = 4\pi R^2 \frac{\dot{M} v_{WIND}^2}{4\pi R^2 v_{WIND}}$$

$$\Rightarrow \frac{R}{t}\frac{d}{dt}\left(R^3 \frac{dR}{dt}\right) = \frac{3\dot{M} v_{WIND}}{4\pi \rho_0}$$

$$\Rightarrow R(t) = \left(\frac{3\dot{M} v_{WIND}}{2\pi \rho_0}\right)^{1/4} t^{1/2}$$

Slower!