Numerical viscosity

rewriting the Lax scheme in a form
FTCS + remainder

$$\frac{q_j^{n+1}-q_j^n}{\Delta t} = -v\left(\frac{q_{j+1}^n-q_{j-1}^n}{2\Delta x}\right)+\underbrace{\frac{1}{2}\left(\frac{q_{j+1}^n-2q_j^n+q_{j-1}^n}{\Delta t}\right)}$$

$$\frac{\partial^2 q}{\partial x^2} = \frac{\frac{q_{j+1}-q_j}{\Delta x}-\frac{q_j-q_{j-1}}{\Delta x}}{\Delta x} = \frac{q_{j+1}-2q_j+q_{j-1}}{\Delta x^2}$$

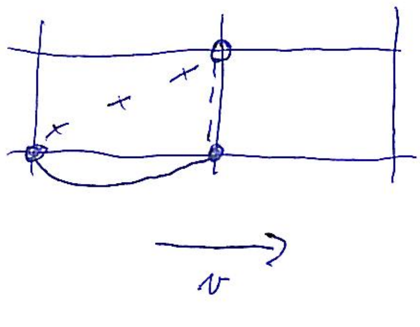
$\Rightarrow$ equivalent to FTCS discretization of the eq.

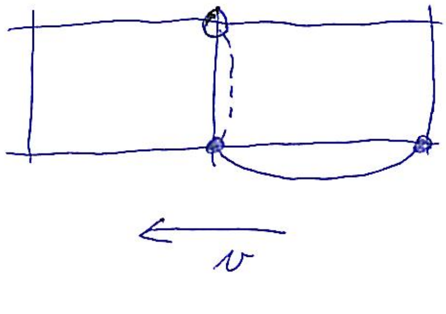
$$\frac{\partial q}{\partial t}+v\frac{\partial q}{\partial x}-\underbrace{\frac{\Delta x^2}{2\Delta t}\frac{\partial^2 q}{\partial x^2}}_{\text{viscous term}}$$

Upwind differencing

- centered differencing takes info from regions which the flow hasn't reach yet

$$\frac{q_j^{n+1}-q_j^n}{\Delta t} = -v\begin{cases}\dfrac{q_j^n-q_{j-1}^n}{\Delta x} & \text{for } v>0\\[2ex] \dfrac{q_{j+1}^n-q_j^n}{\Delta x} & \text{for } v<0\end{cases}$$





- more stable for supersonic flows (Godunov, 1959)